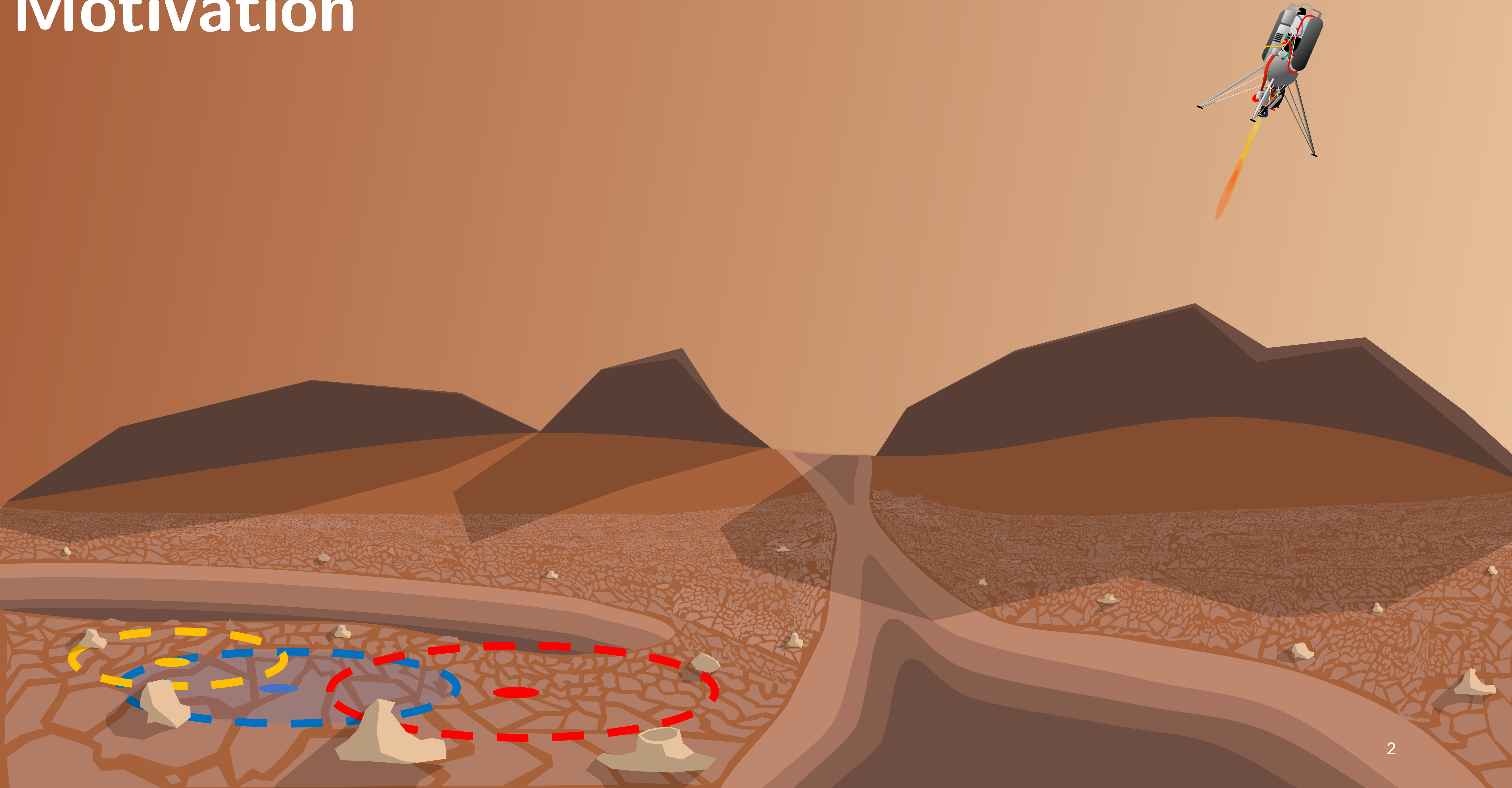
Agile and Aware: Joint Perception and Motion Planning for Highly Dynamic Autonomous Systems

Chris Hayner

January 9, 2026

Motivation



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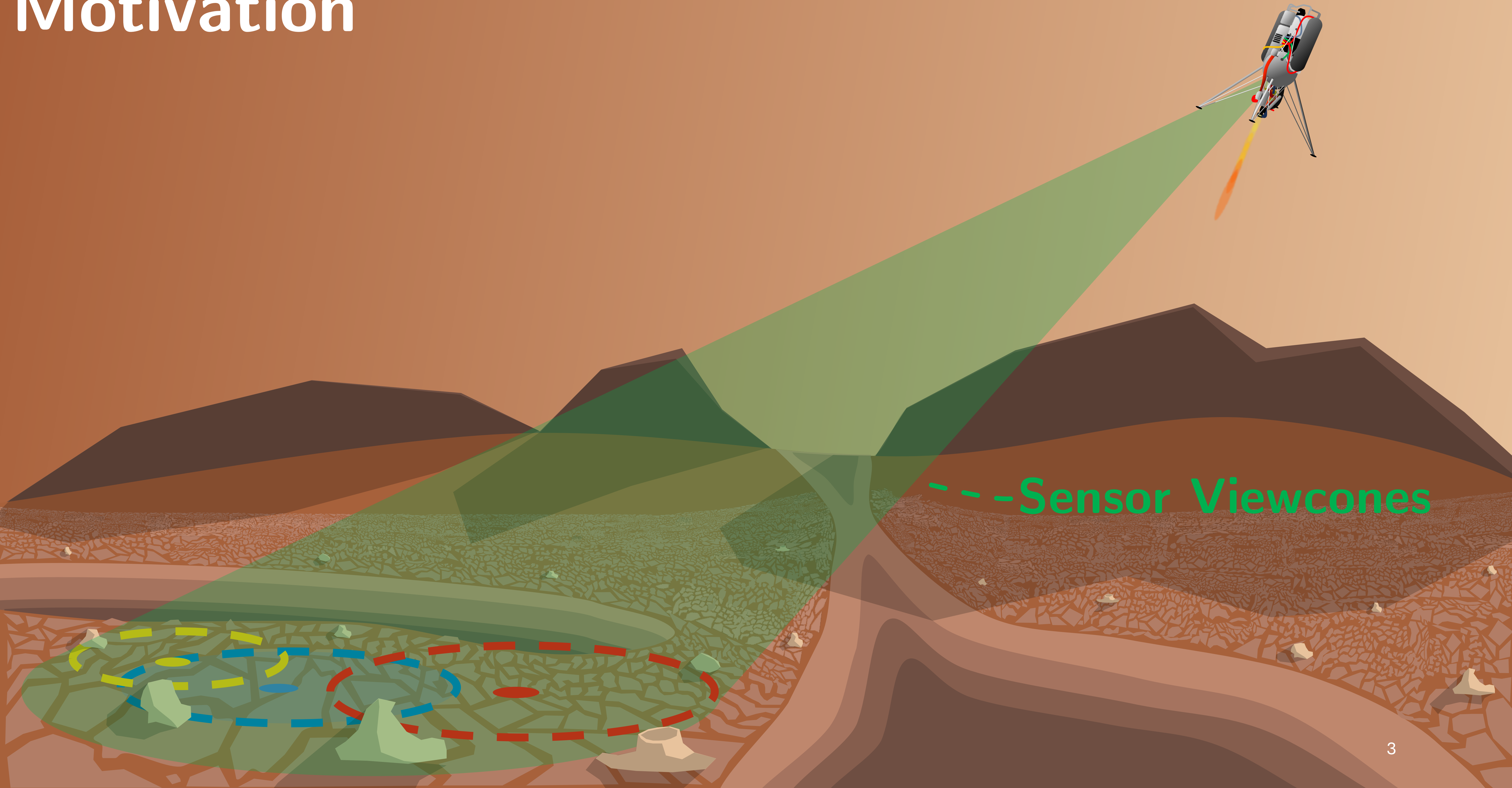
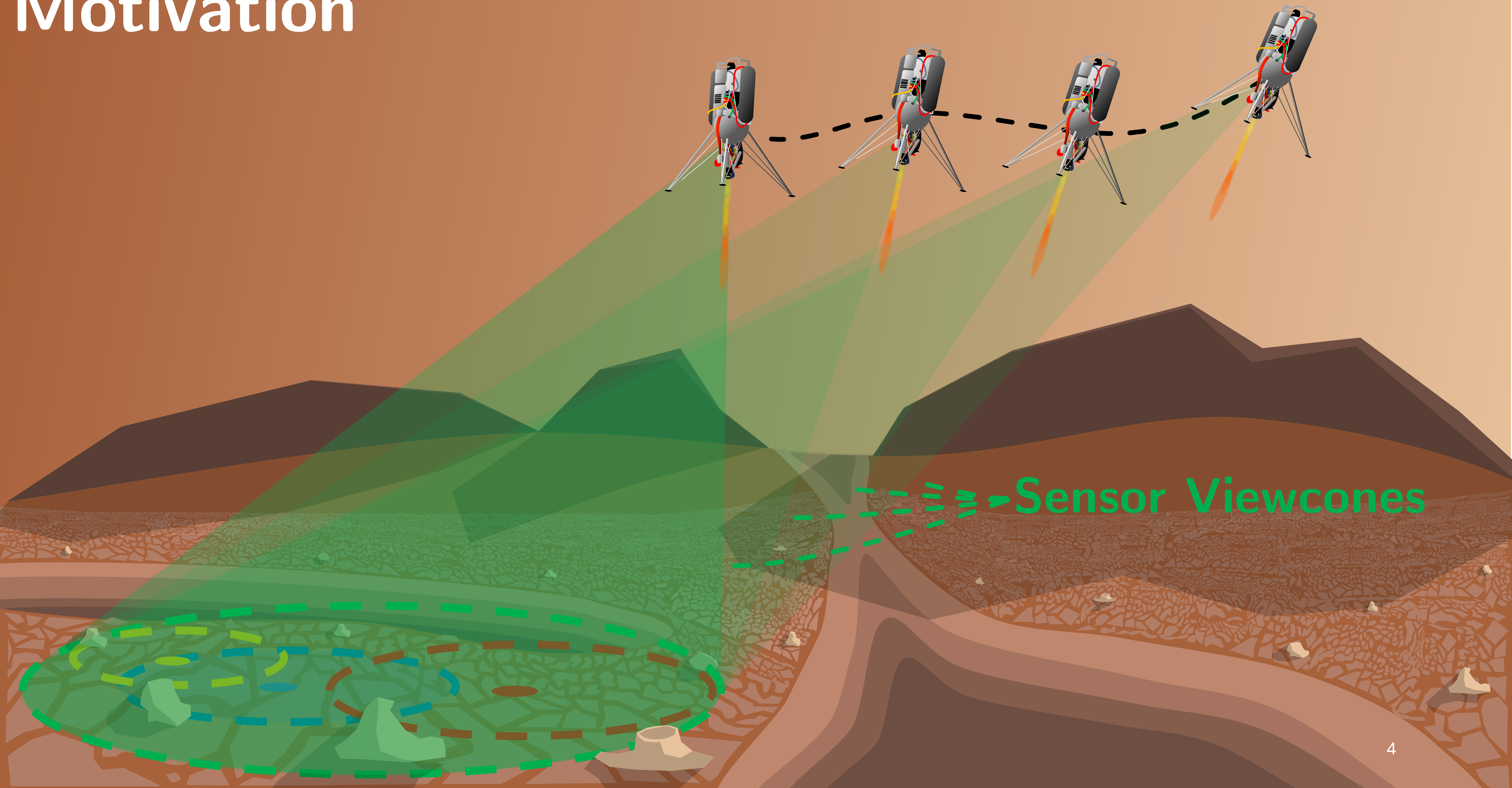


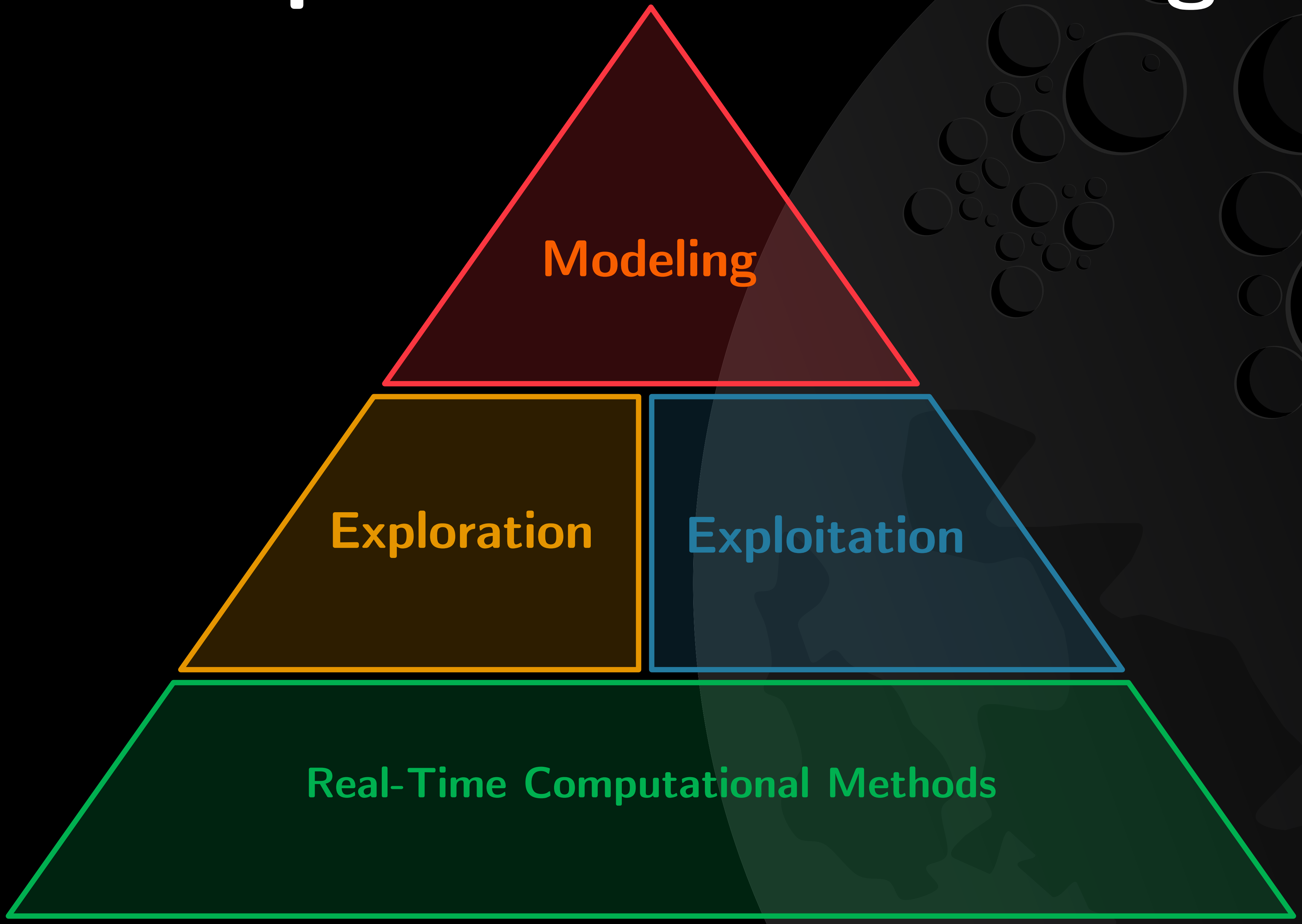
Motivation



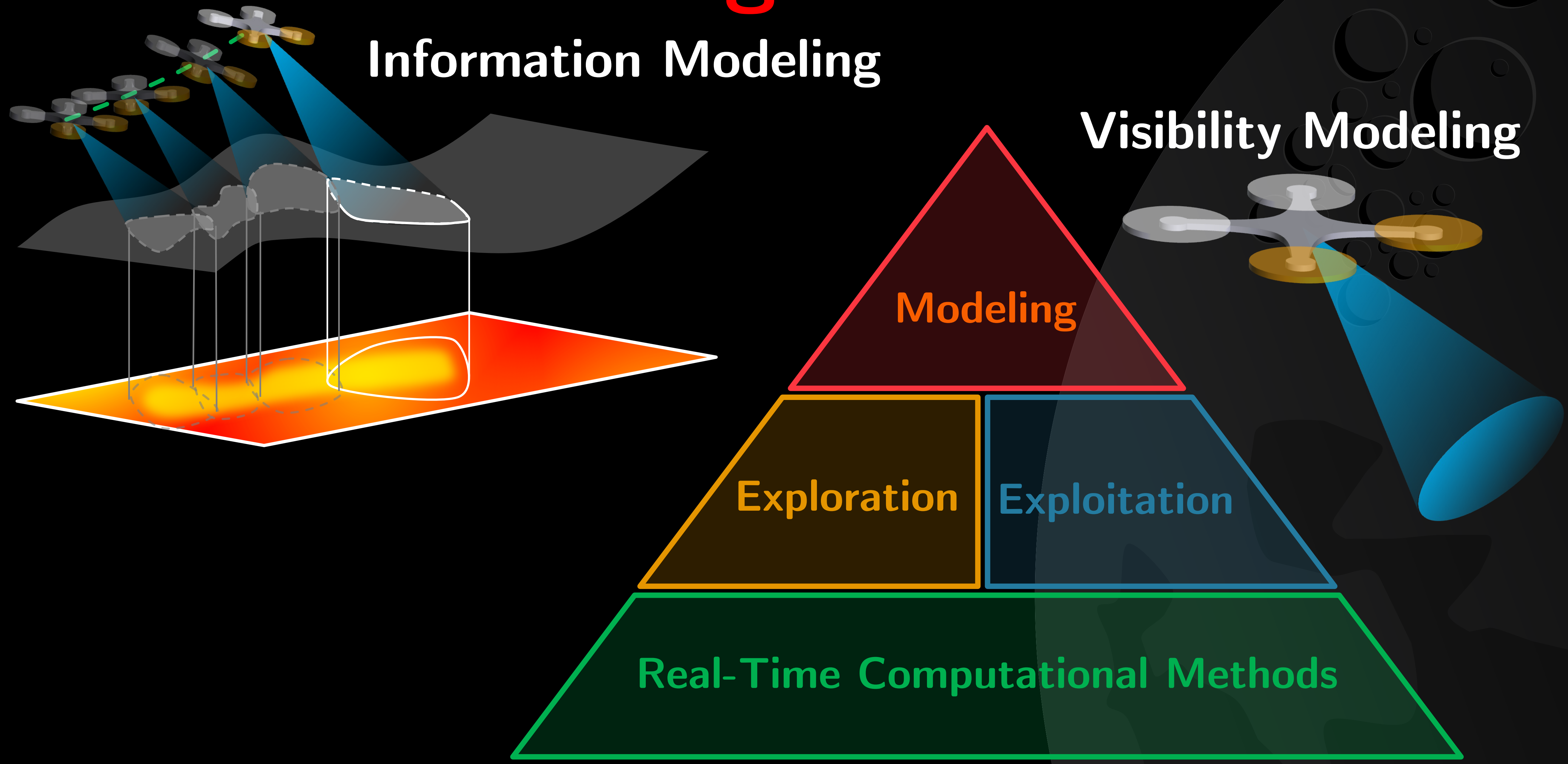
Motivation

[Hayner 2023]

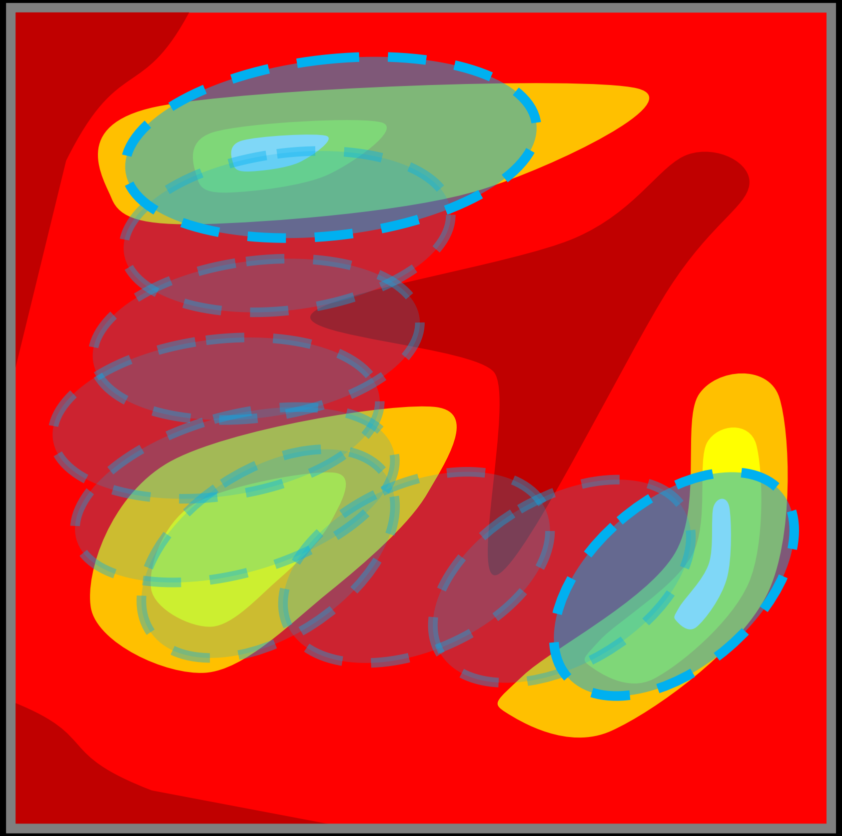
Context: Perception-Aware Planning



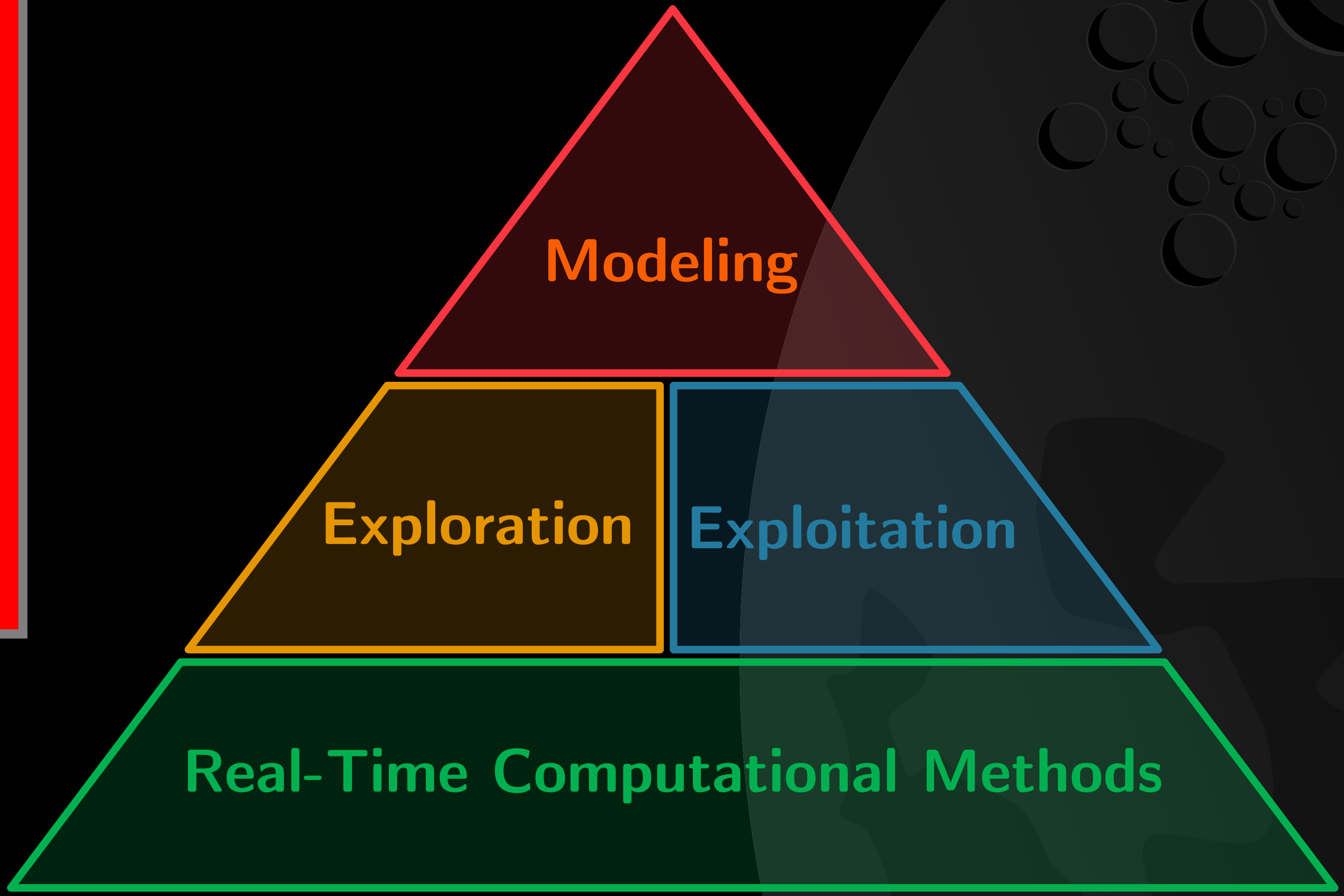
Context: Modeling



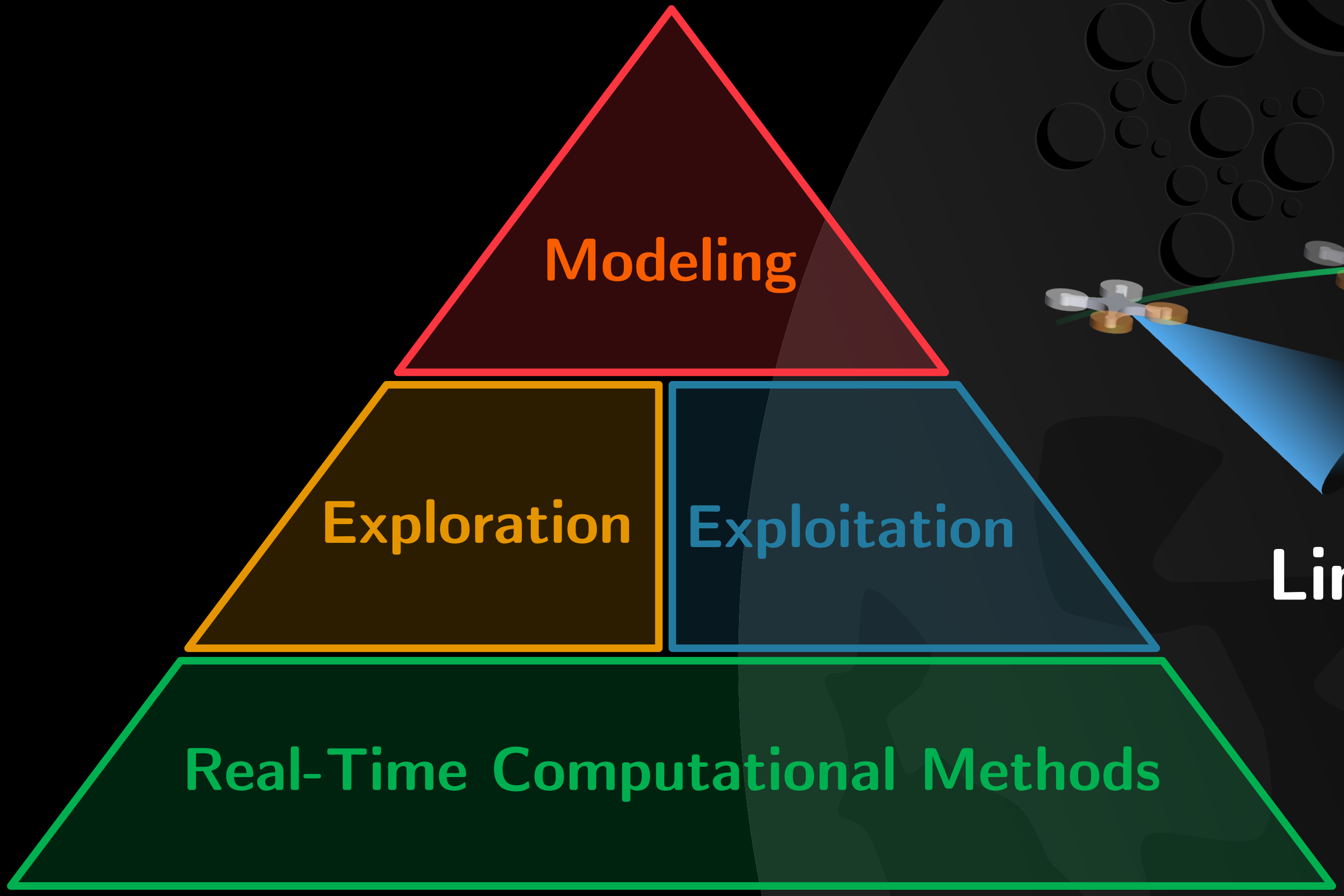
Context: Exploration

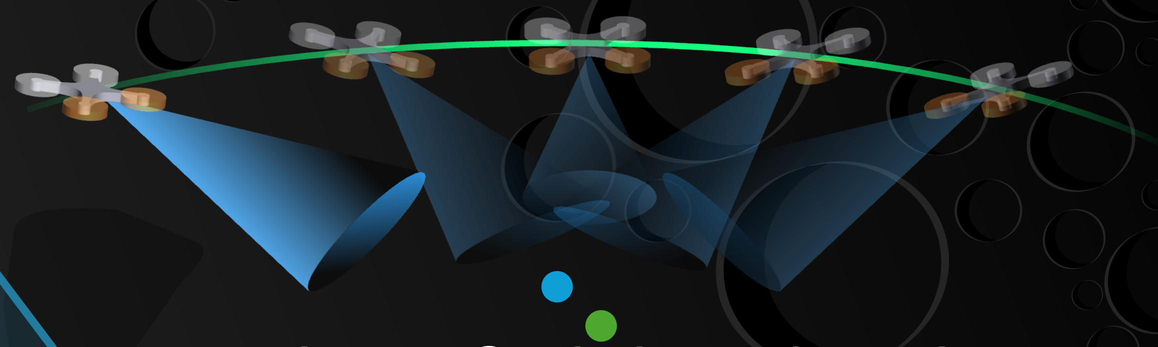


Coverage Planning

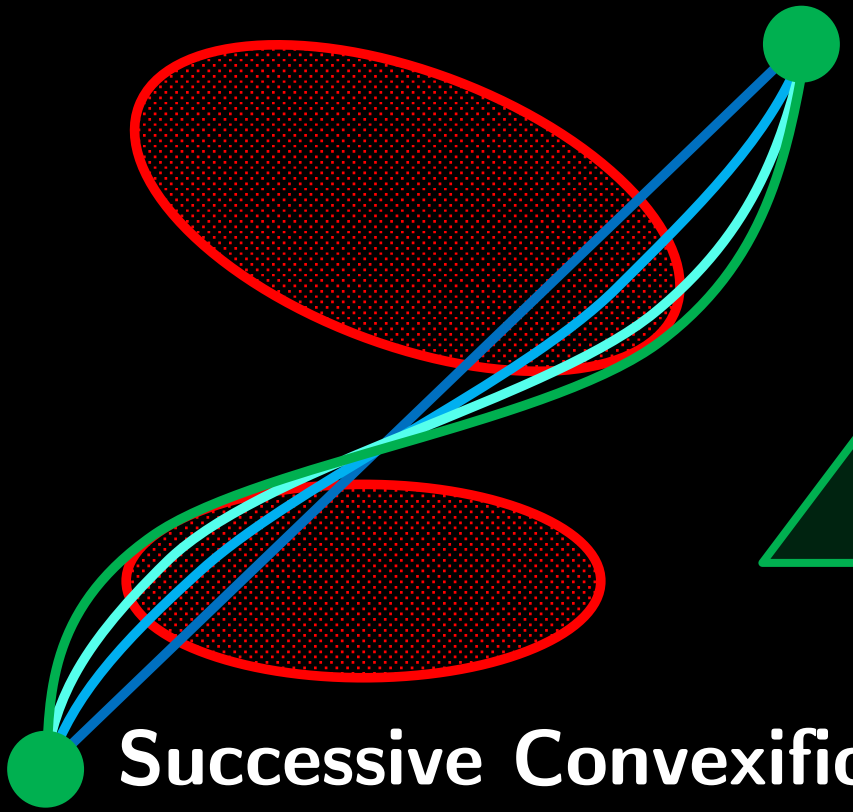


Context: Exploitation

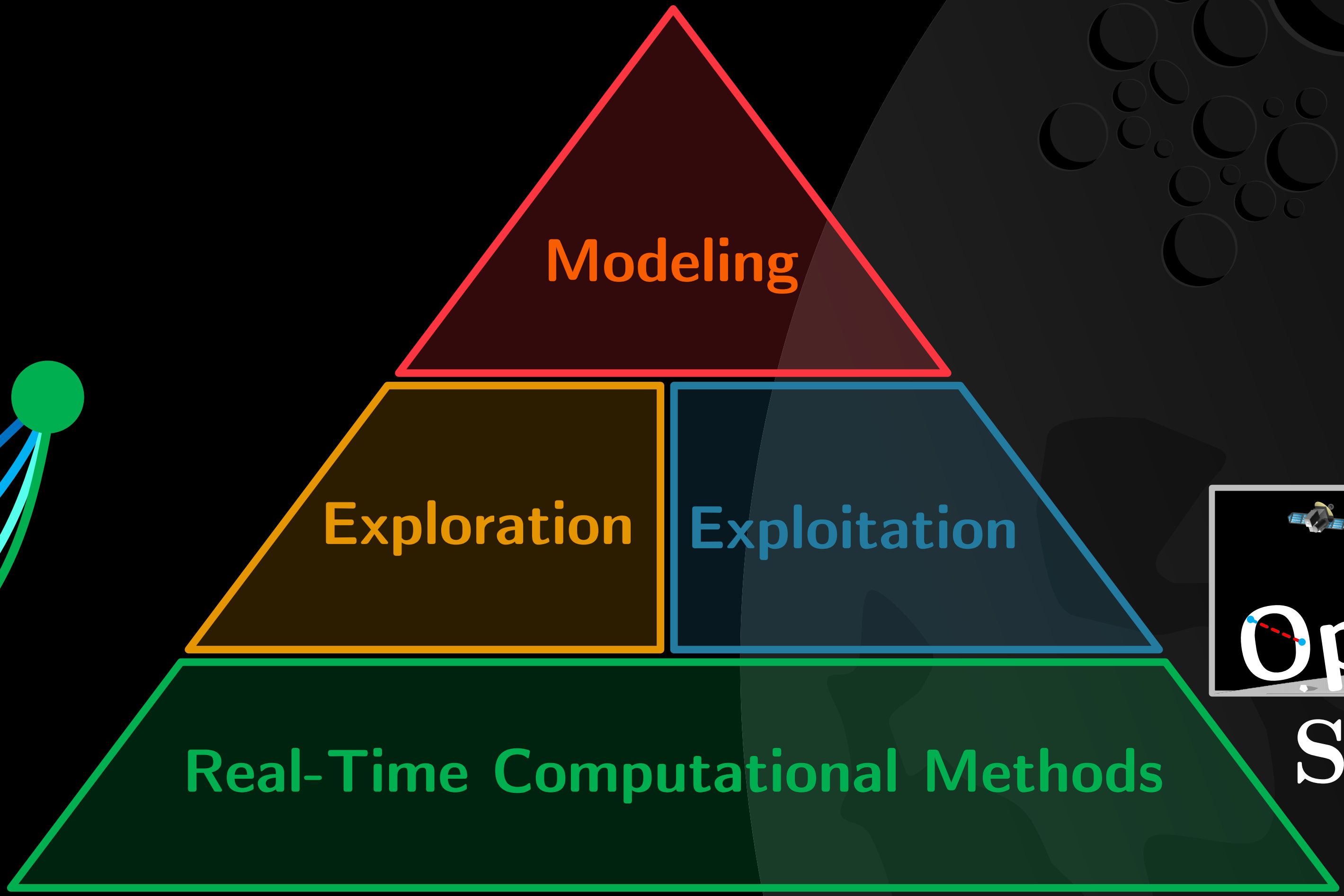




Context: Real-Time Computational Methods



Successive Convexification



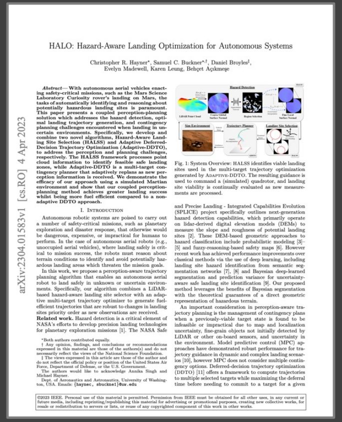
SCvxGEN

Nonconvex Trajectory Optimization Solvers

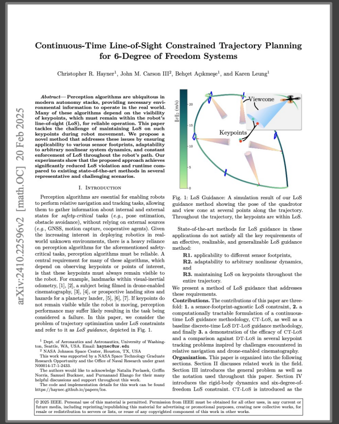
Current Relevant Papers


Information-Aware Powered Descent Guidance for Entry, Descent and Landing

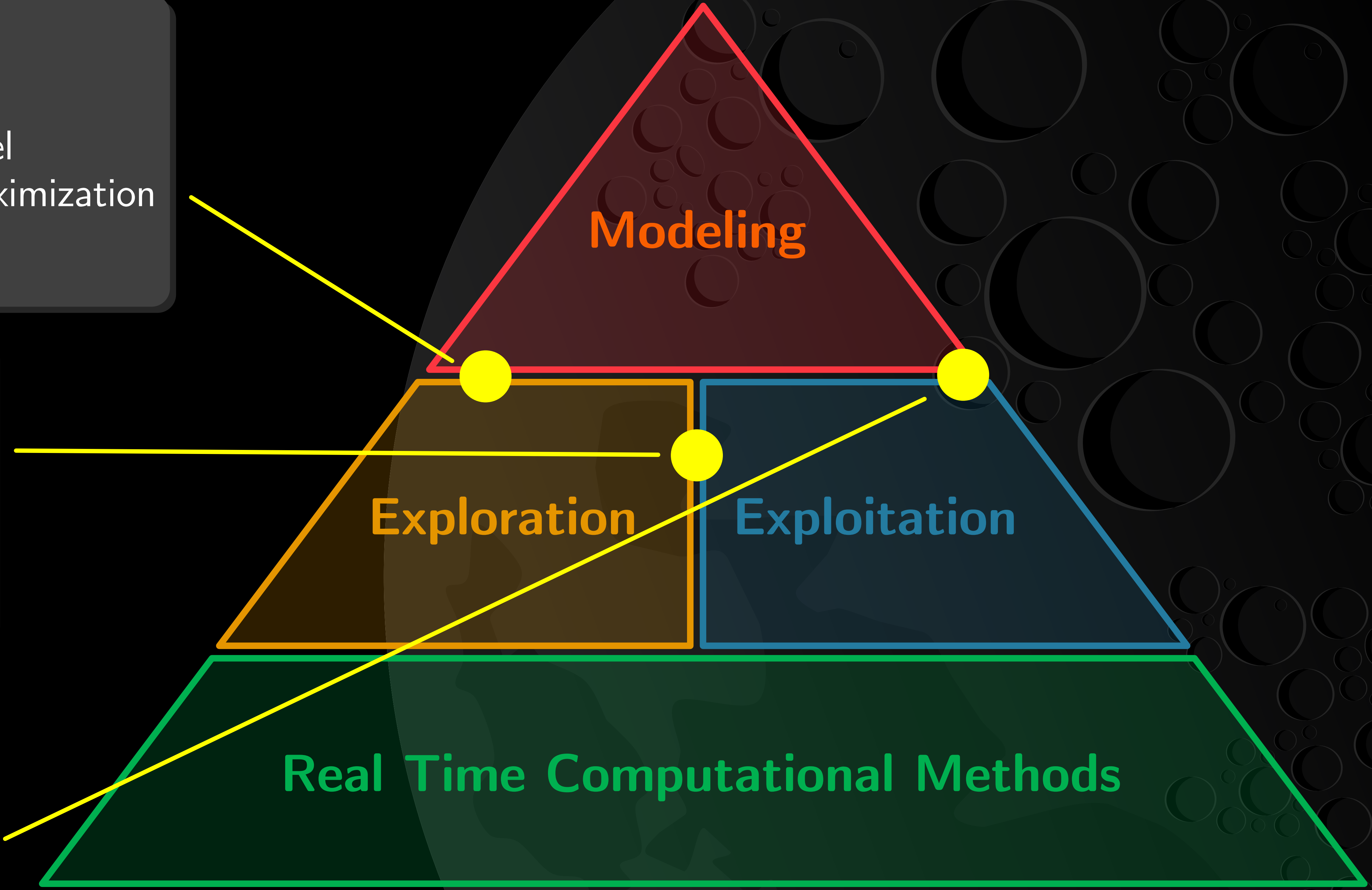
SciTech 2025
- Information Rate Model
- Direct Information Maximization


HALO: Hazard-Aware Landing Optimization for Autonomous Systems

ICRA 2023
- Hazard Modeling
- Multitarget Trajectory Optimization


Continuous-Time Line-of-Sight Constrained Trajectory Planning for 6-Degree of Freedom Systems

RA-L 2025
- Visibility Model
- Exploitation Methods



Preliminaries: Frames

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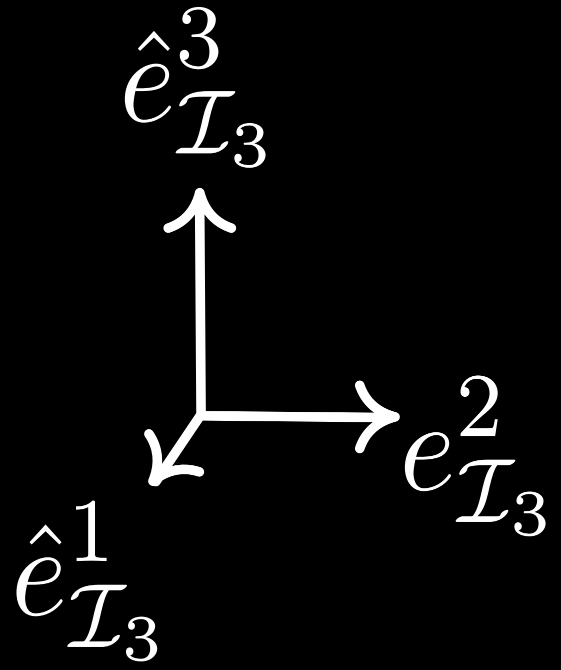
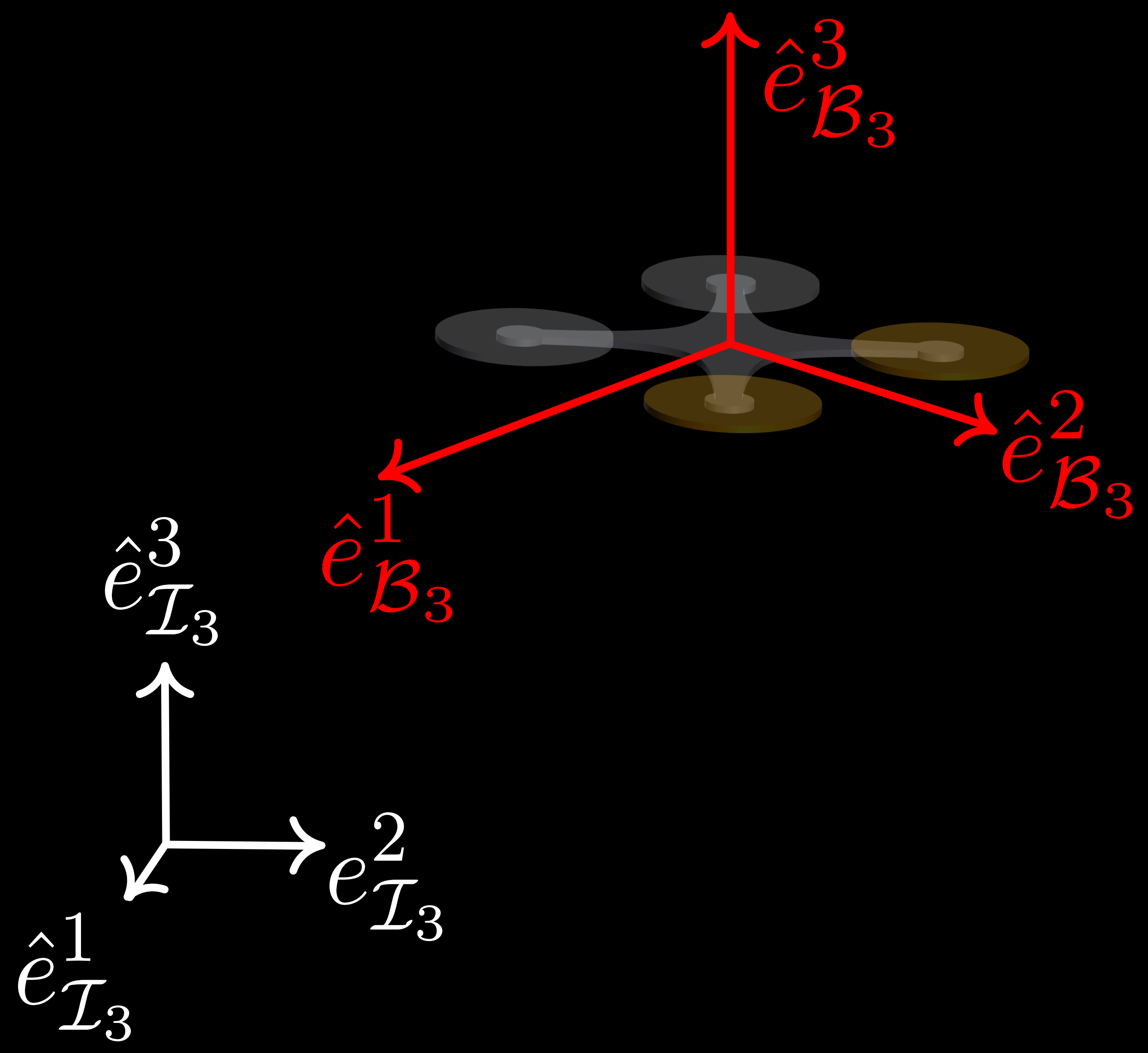


$\mathcal{I}_N$ - Inertial Frame,

in $\mathbb{R}^N$.

Preliminaries: Frames

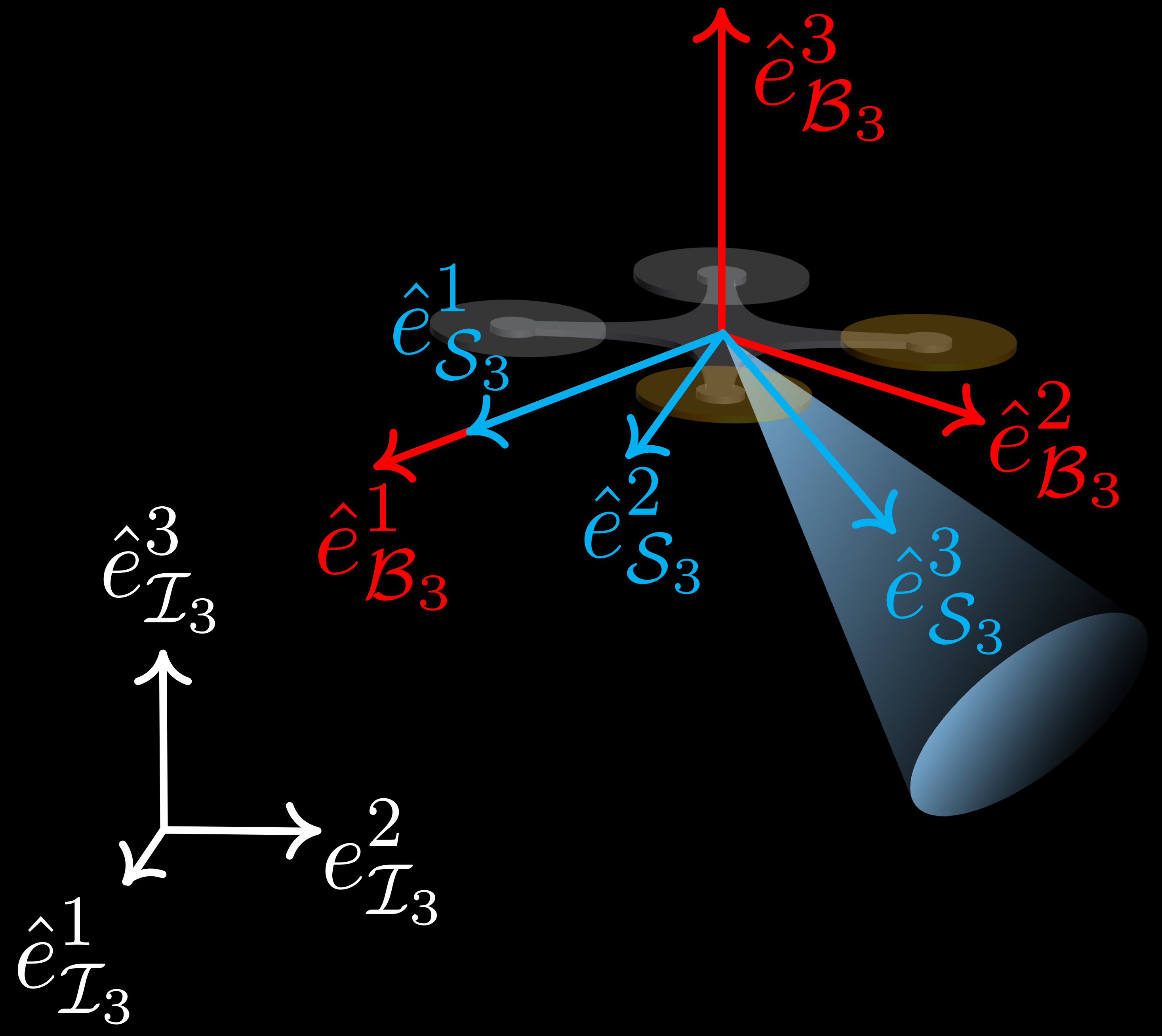


$\mathcal{I}_N$ - Inertial Frame,

$\mathcal{B}_N$ - Body frame,

in $\mathbb{R}^N$.

Preliminaries: Frames



$\mathcal{I}_N$ - Inertial Frame,

$\mathcal{B}_N$ - Body frame,

$\mathcal{S}_N$ - Sensor frame,

in $\mathbb{R}^N$.

Preliminaries: Optimal Control

Definition: Bolza Form

Running Cost **Terminal Cost**

$$
\begin{aligned}
\min_{x,u,t_f} \quad & \int_{t_0}^{t_f} L_r(t, x(t), u(t))\, dt + L(t_f, x(t_f)) \\
\text{subject to} \quad & \dot{x}(t) = f(t, x(t), u(t)), \quad t \in [t_0, t_f] \\
& g(t, x(t), u(t)) \leq 0_{n_g}, \quad t \in [t_0, t_f] \\
& h(t, x(t), u(t)) = 0_{n_h}, \quad t \in [t_0, t_f] \\
& P(t_0, x(t_0), t_f, x(t_f)) \leq 0_{n_P} \\
& Q(t_0, x(t_0), t_f, x(t_f)) = 0_{n_Q}
\end{aligned}
$$

[Elango 2025]

Preliminaries: Optimal Control

Definition: Bolza Form

Running Cost — Terminal Cost

$$\min_{x,u,t_f} \int_{t_0}^{t_f} L_r(t, x(t), u(t))\, dt + L(t_f, x(t_f))$$

$$\begin{aligned}
\text{subject to}\quad & \dot{x}(t) = f(t, x(t), u(t)), \quad t \in [t_0, t_f] \quad \textbf{Dynamics}\\
& g(t, x(t), u(t)) \leq 0_{n_g}, \quad t \in [t_0, t_f]\\
& h(t, x(t), u(t)) = 0_{n_h}, \quad t \in [t_0, t_f]\\
& P(t_0, x(t_0), t_f, x(t_f)) \leq 0_{n_P}\\
& Q(t_0, x(t_0), t_f, x(t_f)) = 0_{n_Q}
\end{aligned}$$

[Elango 2025]

Preliminaries: Optimal Control

Definition: Bolza Form

Running Cost / **Terminal Cost**

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\end{aligned}
$$

Dynamics

Inequality Constraints

[Elango 2025]

Preliminaries: Optimal Control

Definition: Bolza Form

Running Cost **Terminal Cost**

$$\min_{x,u,t_f} \int_{t_0}^{t_f} L_r(t, x(t), u(t))\, dt + L(t_f, x(t_f))$$

$$\begin{aligned} \text{subject to} \quad & \dot{x}(t) = f(t, x(t), u(t)), \quad t \in [t_0, t_f] && \textbf{Dynamics} \\ & g(t, x(t), u(t)) \le 0_{n_g}, \quad t \in [t_0, t_f] && \textbf{Inequality Constraints} \\ & h(t, x(t), u(t)) = 0_{n_h}, \quad t \in [t_0, t_f] && \textbf{Equality Constraints} \\ & P(t_0, x(t_0), t_f, x(t_f)) \le 0_{n_P} \\ & Q(t_0, x(t_0), t_f, x(t_f)) = 0_{n_Q} \end{aligned}$$

[Elango 2025]

Preliminaries: Optimal Control

Definition: Bolza Form

Running Cost — Terminal Cost

$$\min_{x,u,t_f} \int_{t_0}^{t_f} L_r(t, x(t), u(t))\, dt + L(t_f, x(t_f))$$

$$\text{subject to} \quad \dot{x}(t) = f(t, x(t), u(t)), \quad t \in [t_0, t_f]$$ Dynamics

$$g(t, x(t), u(t)) \leq 0_{n_g}, \quad t \in [t_0, t_f]$$ Inequality Constraints

$$h(t, x(t), u(t)) = 0_{n_h}, \quad t \in [t_0, t_f]$$ Equality Constraints

$$P(t_0, x(t_0), t_f, x(t_f)) \leq 0_{n_P}$$ Inequality Boundary Constraints

$$Q(t_0, x(t_0), t_f, x(t_f)) = 0_{n_Q}$$

[Elango 2025]

Preliminaries: Optimal Control

Definition: Bolza Form

$$\min_{x,u,t_f} \quad \underbrace{\int_{t_0}^{t_f} L_r(t, x(t), u(t))\, dt}_{\text{Running Cost}} + \underbrace{L(t_f, x(t_f))}_{\text{Terminal Cost}}$$

$$\begin{aligned}
\text{subject to} \quad & \dot{x}(t) = f(t, x(t), u(t)), \quad t \in [t_0, t_f] && \text{Dynamics} \\
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\end{aligned}$$

[Elango 2025]

Preliminaries: Optimal Control

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$$

[Liberzon 2011]

Preliminaries: Optimal Control

Bolza Form

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$$\dot{\xi} = L_r(t, x(t), u(t)), \quad \xi(t_0) = 0,$$

$$\dot{\tilde{x}} = \begin{bmatrix} f(t, x(t), u(t)) \\ L_r(t, x(t), u(t)) \end{bmatrix} = \tilde{f}(t, x(t), u(t))$$

[Liberzon 2011]

Preliminaries: Optimal Control

Bolza Form

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$$\int_{t_0}^{t_f} L_r(t,x(t),u(t))\,dt = \xi(t_f)$$

[Liberzon 2011]

Preliminaries: Optimal Control

Mayer Form

$$\begin{aligned}
\min_{x,u,t_f} \quad & \xi(t_f) + L(t_f, x(t_f)) \\
\text{subject to} \quad & \dot{\tilde{x}}(t) = \tilde{f}(t, x(t), u(t)), \quad t \in [t_0, t_f] \\
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[Liberzon 2011]

Preliminaries: Optimal Control

Mayer Form

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& Q(t_0, x(t_0), t_f, x(t_f)) = 0_{n_Q}
\end{aligned}
$$

It's desirable to express general problems in this form as the majority effort of gradient computation is consolidated in one place, leading to efficient numerical implementations.

[Elango 2025]

Preliminaries: Optimal Control

Solvers for Nonlinear Programs (NLP's) expressed in the Mayer Form can be split up into the following:

Direct NLP Solvers

- IPOPT **[Wachter 2006]**
- SNOPT **[Gill 2006]**
- Knitro **[Byrd 2006]**

Sequential Convex Programming

- GuSTO **[Bonalli 2019]**
- SCvx **[Malyuta 2022]**
- CT-SCvx **[Elango 2025]**

Preliminaries: Optimal Control

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Shortcomings:
- Lack convergence guarantees
- Require 2nd Order Information
- Unsuitable for real-time applications
- No or limited continuous time constraint satisfaction

Sequential Convex Programming

GuSTO **[Bonalli 2019]**
SCvx **[Malyuta 2022]**
CT-SCvx **[Elango 2025]**

Shortcomings:
- Requires decent initial guess
- Many tuning parameters
- Requires models to be locally near linear

Preliminaries: Optimal Control

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Knitro **[Byrd 2006]**

Shortcomings:
- Lack convergence guarantees
- Require 2nd Order Information
- Unsuitable for real-time applications
- No or limited continuous time constraint satisfaction

Benefits:
- Less Tuning
- Higher accuracy from 2nd Order info

Sequential Convex Programming

GuSTO **[Bonalli 2019]**
SCvx **[Malyuta 2022]**
CT-SCvx **[Elango 2025]**

Shortcomings:
- Requires decent initial guess
- Many tuning parameters
- Requires models to be locally near linear

Benefits:
- Only requires C1 functions
- Can have continuous time constraint satisfaction
- Real-time capable

Preliminaries: Optimal Control

As I am concerned with problems that require real-time computation and continuous time constraint satisfaction, I used CT-SCvx to solve the problems modeled in this work.

Preliminaries: CT-SCvx

Optimal Control Problem

[Elango 2025]

Preliminaries: CT-SCvx

Optimal Control Problem → Path Constraint Reformulation

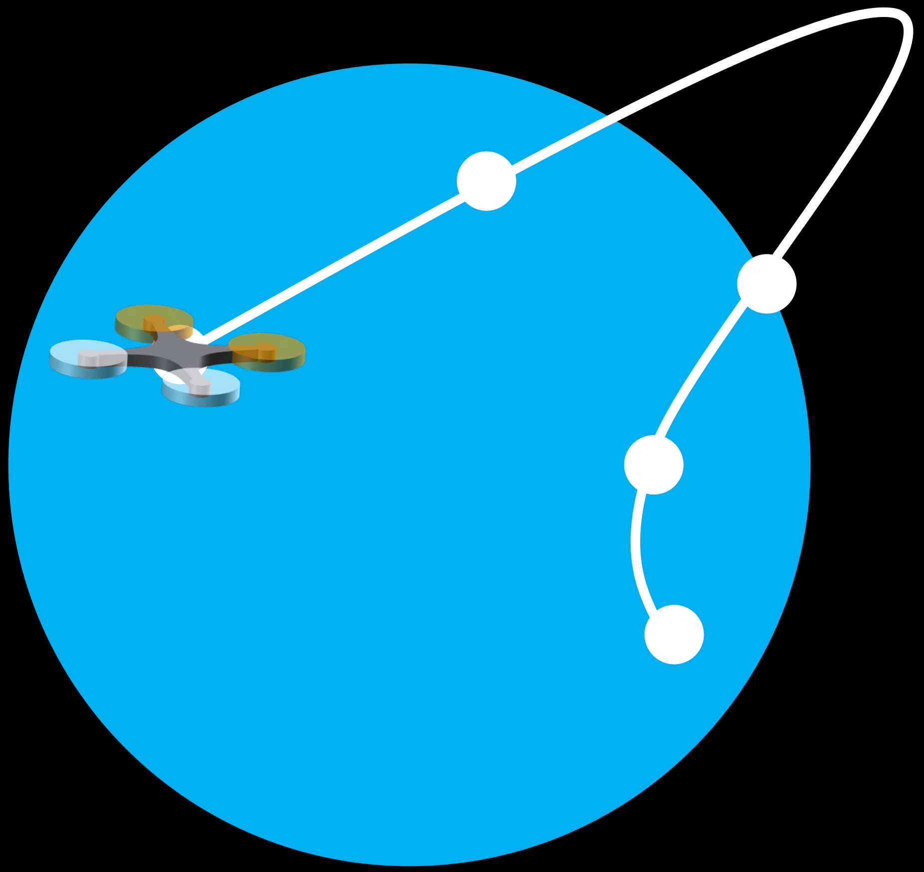
Path Constraints

Discrete Constraints

[Elango 2025]

Preliminaries: CT-SCvx

Optimal Control Problem → Path Constraint Reformulation



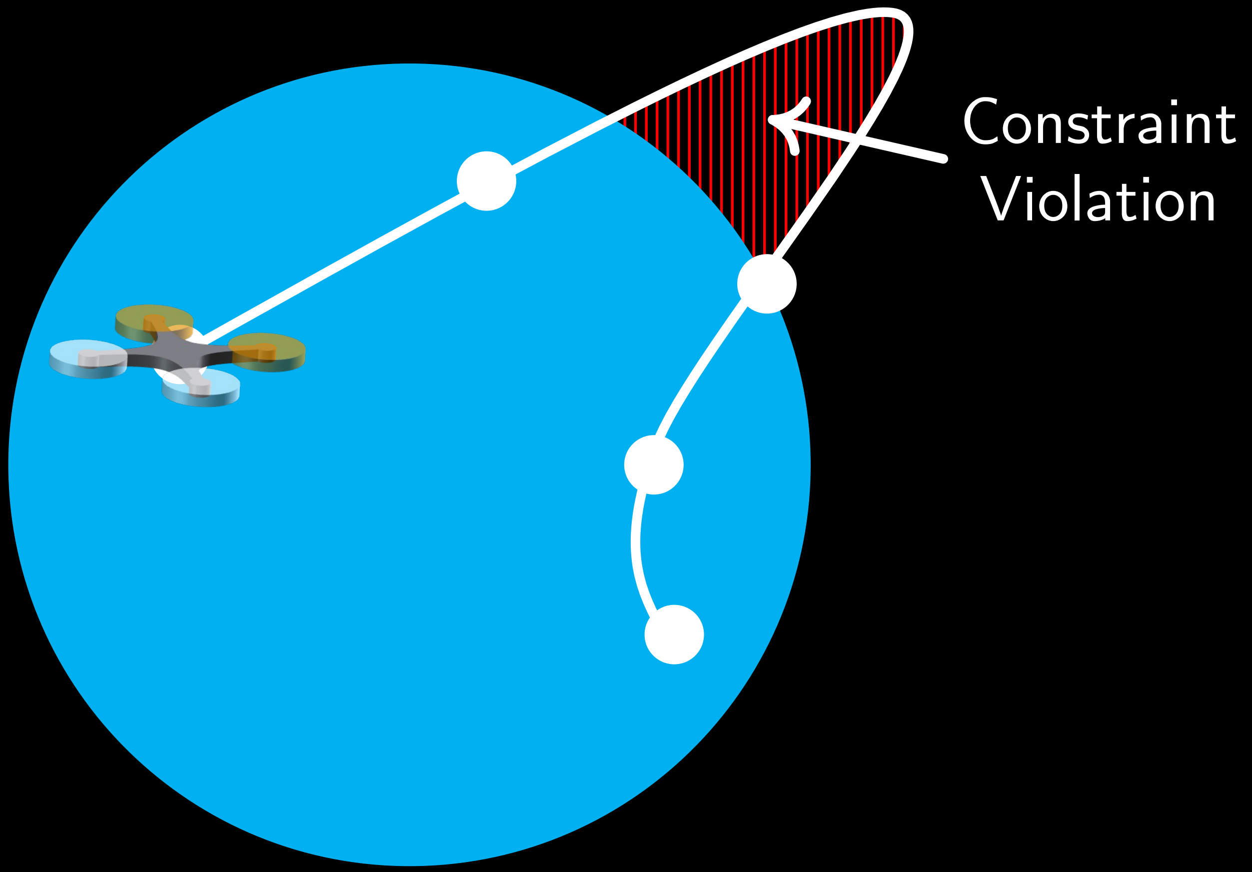
Path Constraints

Discrete Constraints

[Elango 2025]

Preliminaries: CT-SCvx

Optimal Control Problem → Path Constraint Reformulation



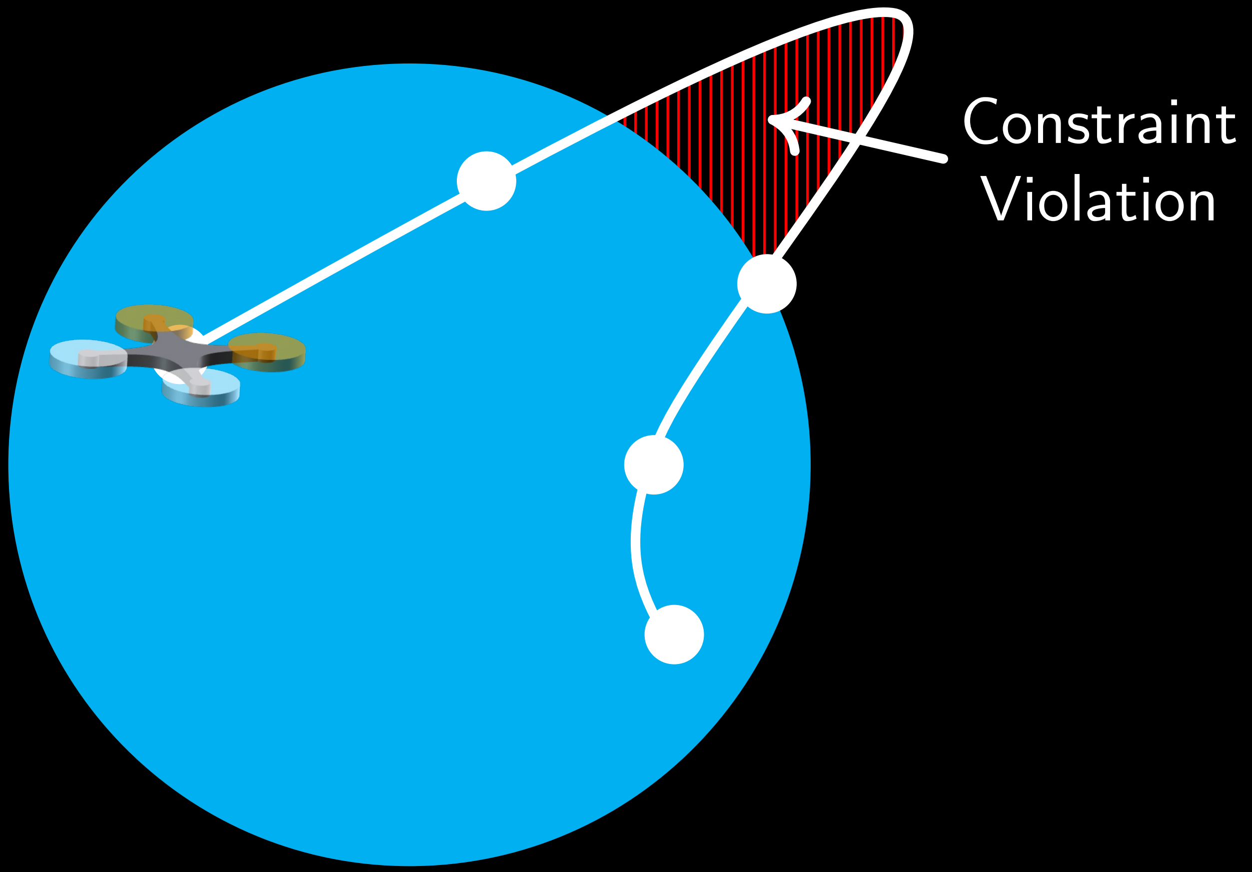
Path Constraints

Discrete Constraints

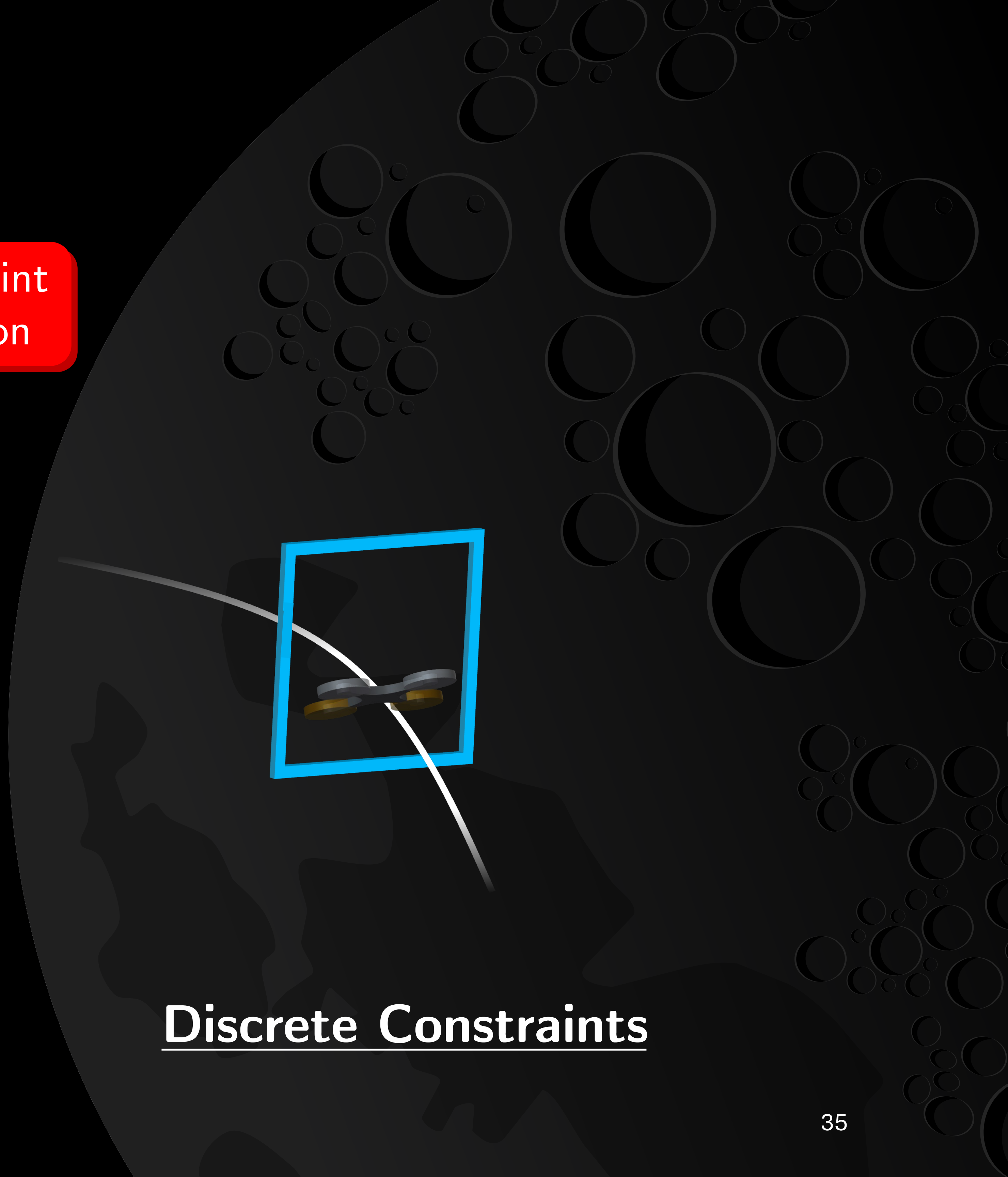
[Elango 2025]

Preliminaries: CT-SCvx

Optimal Control Problem → Path Constraint Reformulation



Path Constraints

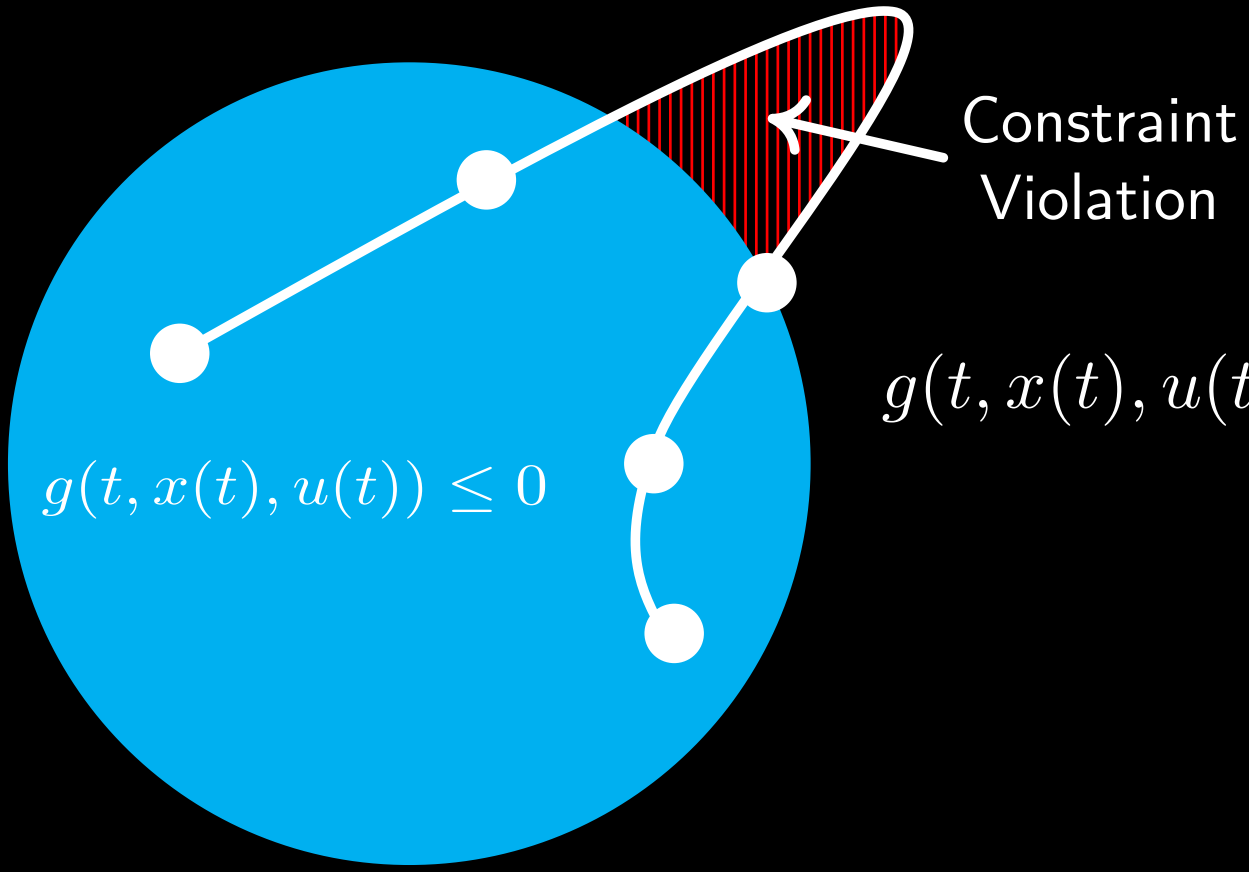


Discrete Constraints

[Elango 2025]

Preliminaries: CT-SCvx

Optimal Control Problem → Path Constraint Reformulation

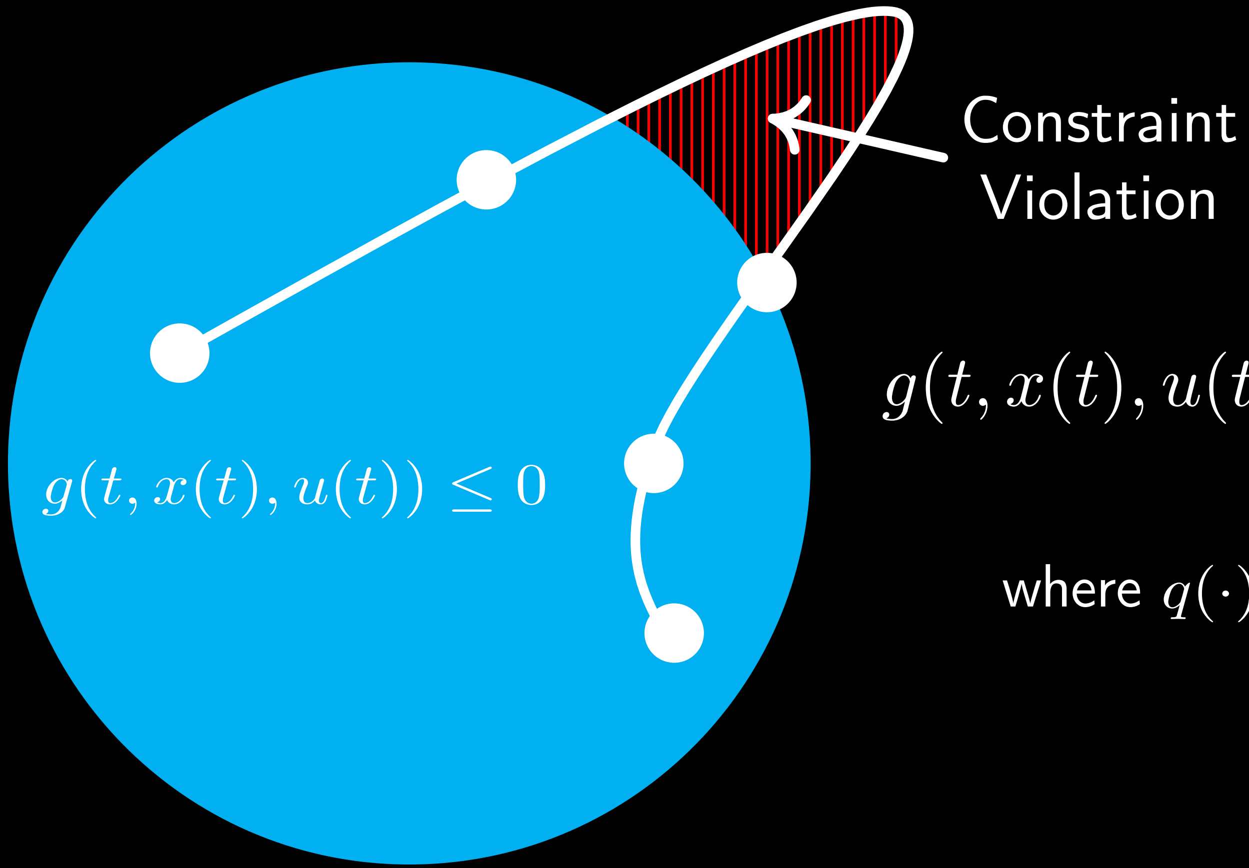


$$g(t, x(t), u(t)) \leq 0 \; \forall t \in [t_0, t_f]$$

[Elango 2025]

Preliminaries: CT-SCvx

Optimal Control Problem → Path Constraint Reformulation

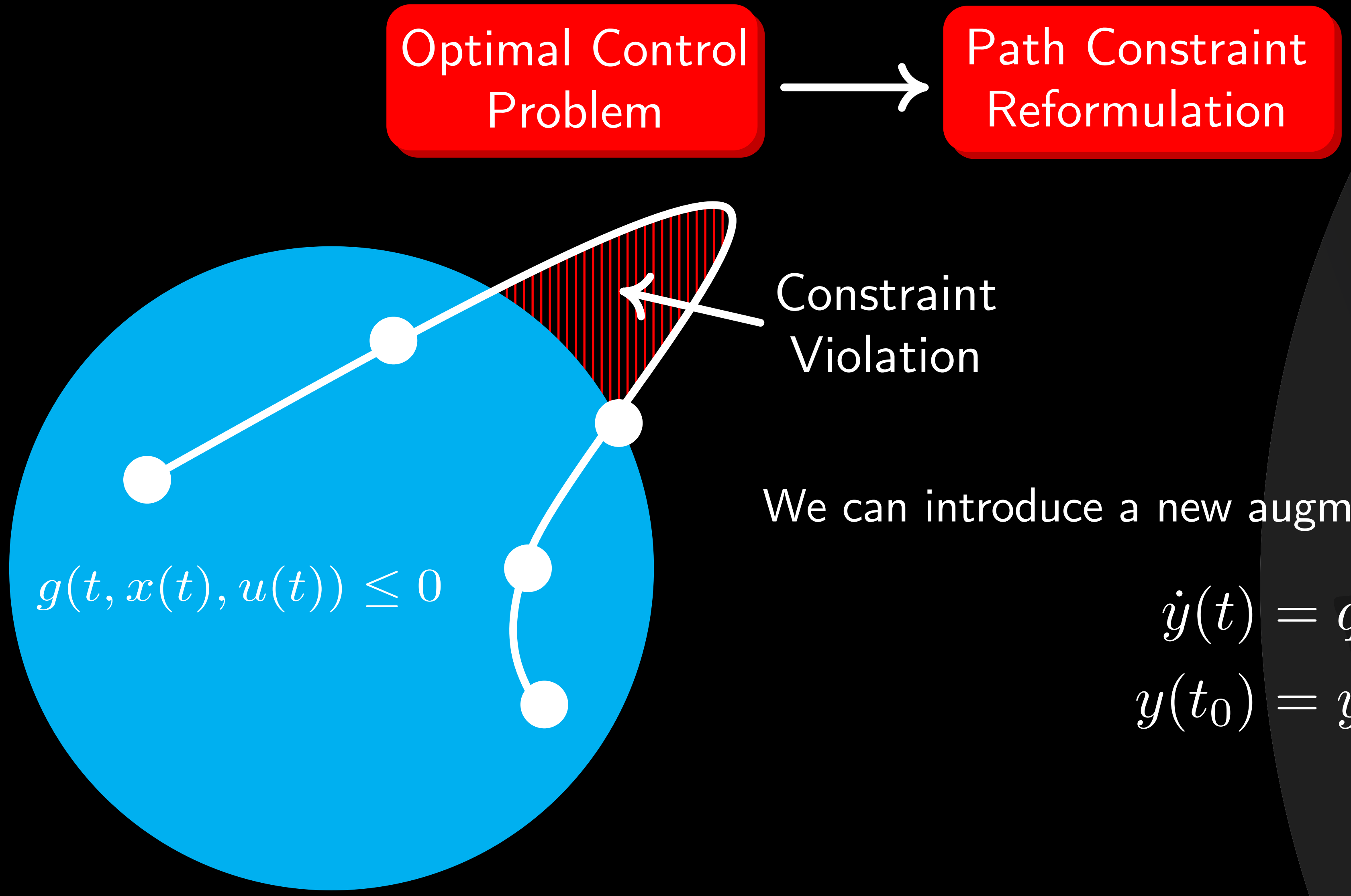


$$g(t, x(t), u(t)) \leq 0 \; \forall t \in [t_0, t_f] \iff \int_{t_0}^{t_f} q(g(t, x(t), u(t)))\, dt = 0$$

where $q(\cdot)$ is the penalty function for $g(\cdot)$

[Elango 2025]

Preliminaries: CT-SCvx

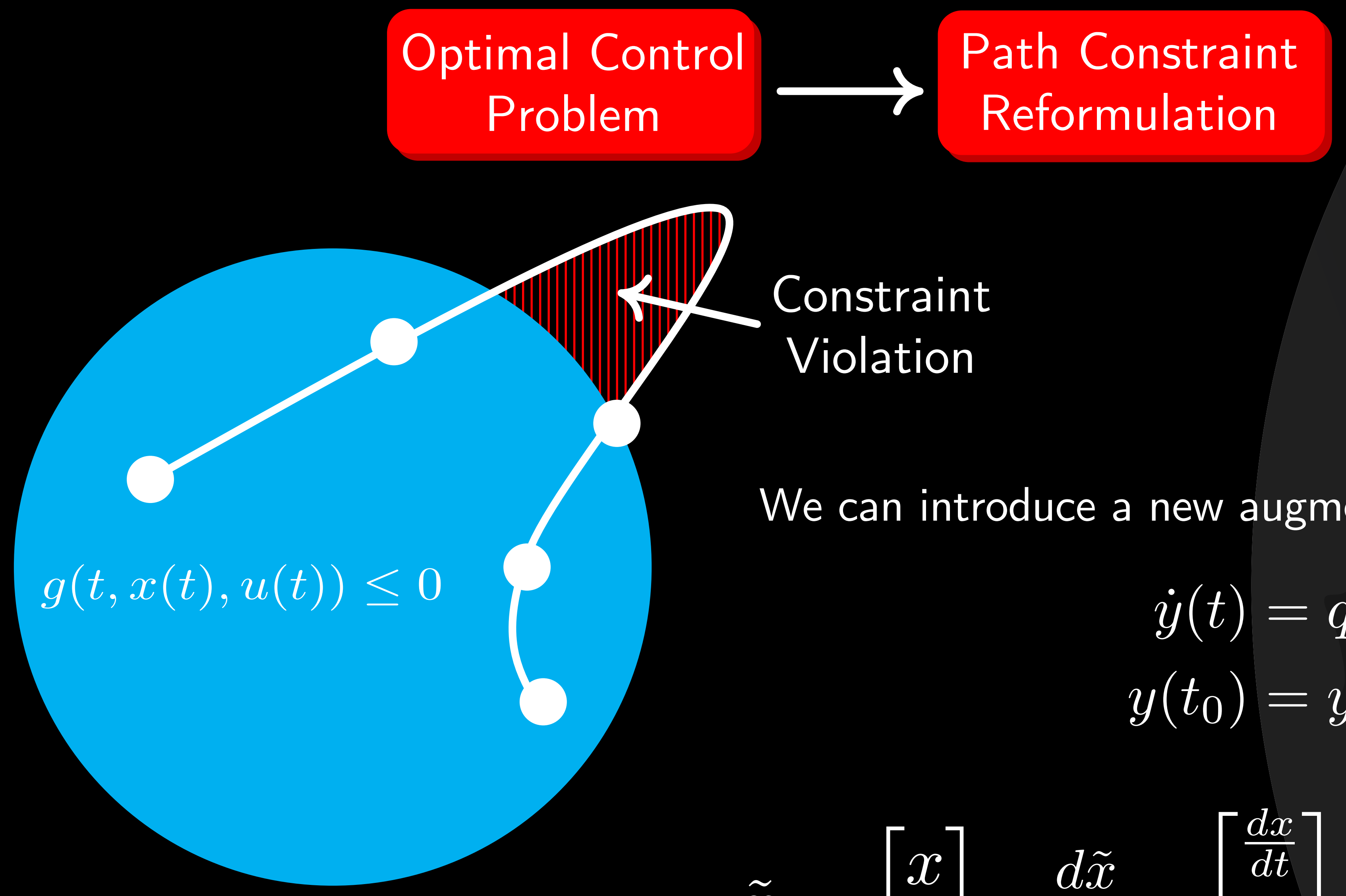


We can introduce a new augmented state,

$$\dot{y}(t) = q(g(t, x(t), u(t)))$$
$$y(t_0) = y(t_f)$$

[Elango 2025]

Preliminaries: CT-SCvx



We can introduce a new augmented state,

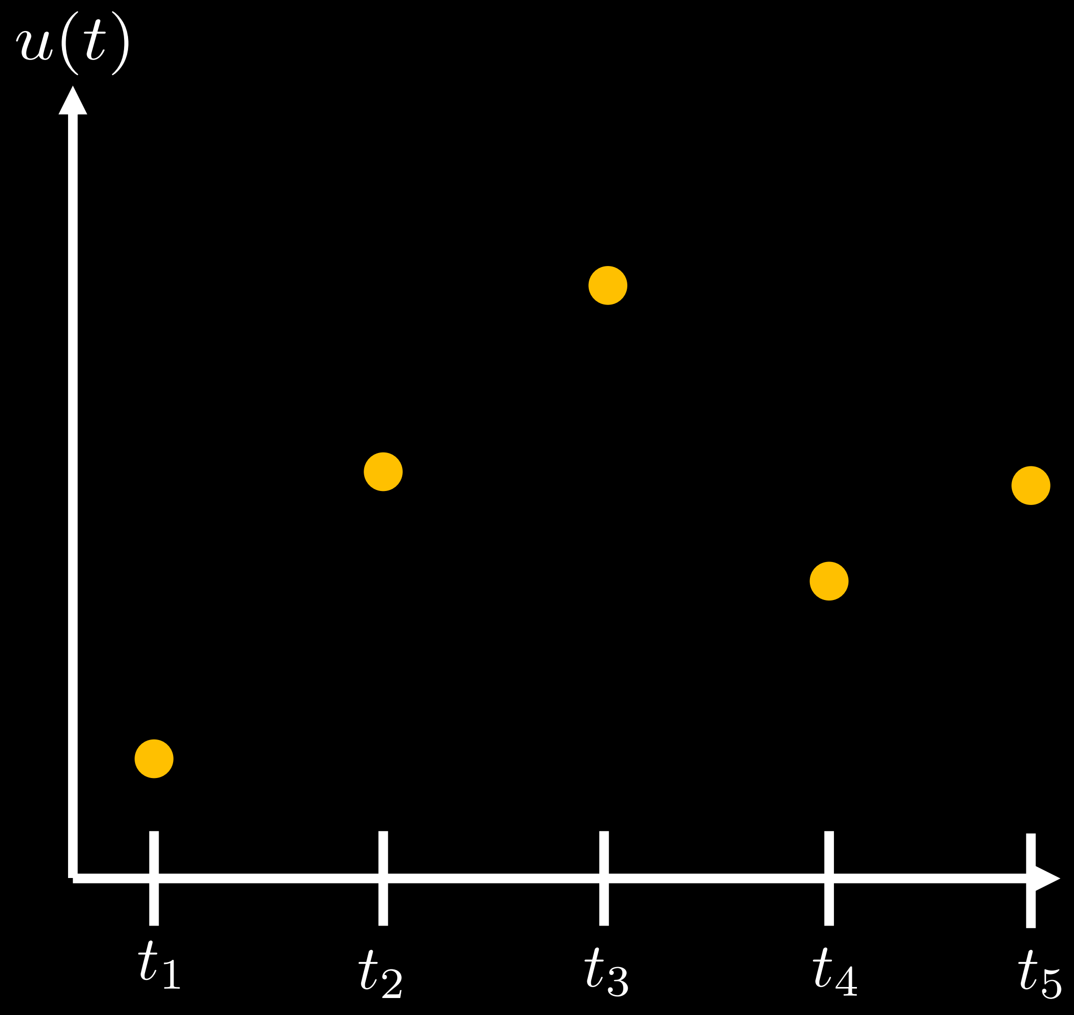
$$\dot{y}(t) = q(g(t, x(t), u(t)))$$
$$y(t_0) = y(t_f)$$

$$\tilde{x} = \begin{bmatrix} x \\ y \end{bmatrix} \quad \frac{d\tilde{x}}{dt} = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \begin{bmatrix} f(x, u) \\ q(g(t, x(t), u(t))) \end{bmatrix}$$

[Elango 2025]

Preliminaries: CT-SCvx

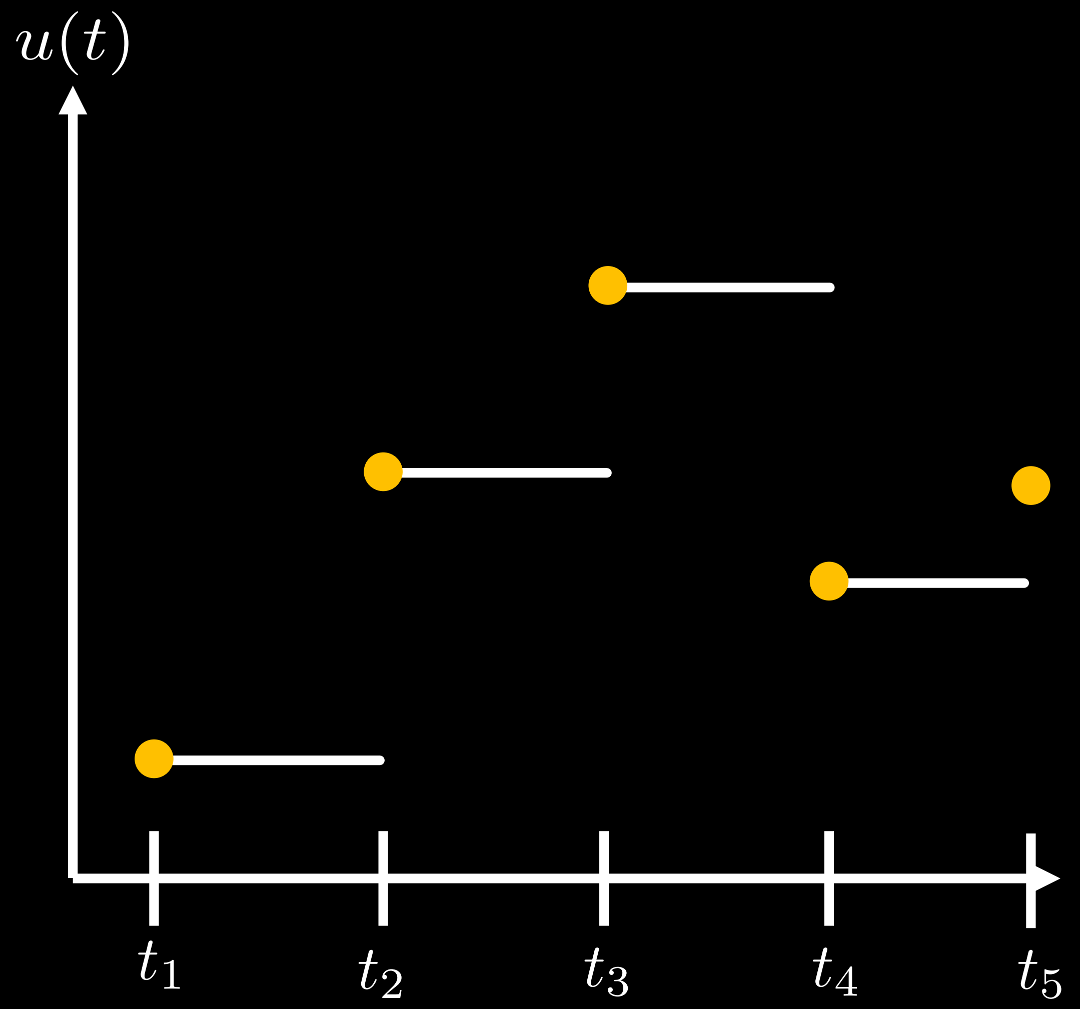
Optimal Control Problem → Path Constraint Reformulation → Parameterization



We need some way to parameterize our control in continuous time.

[Elango 2025]

Preliminaries: CT-SCvx



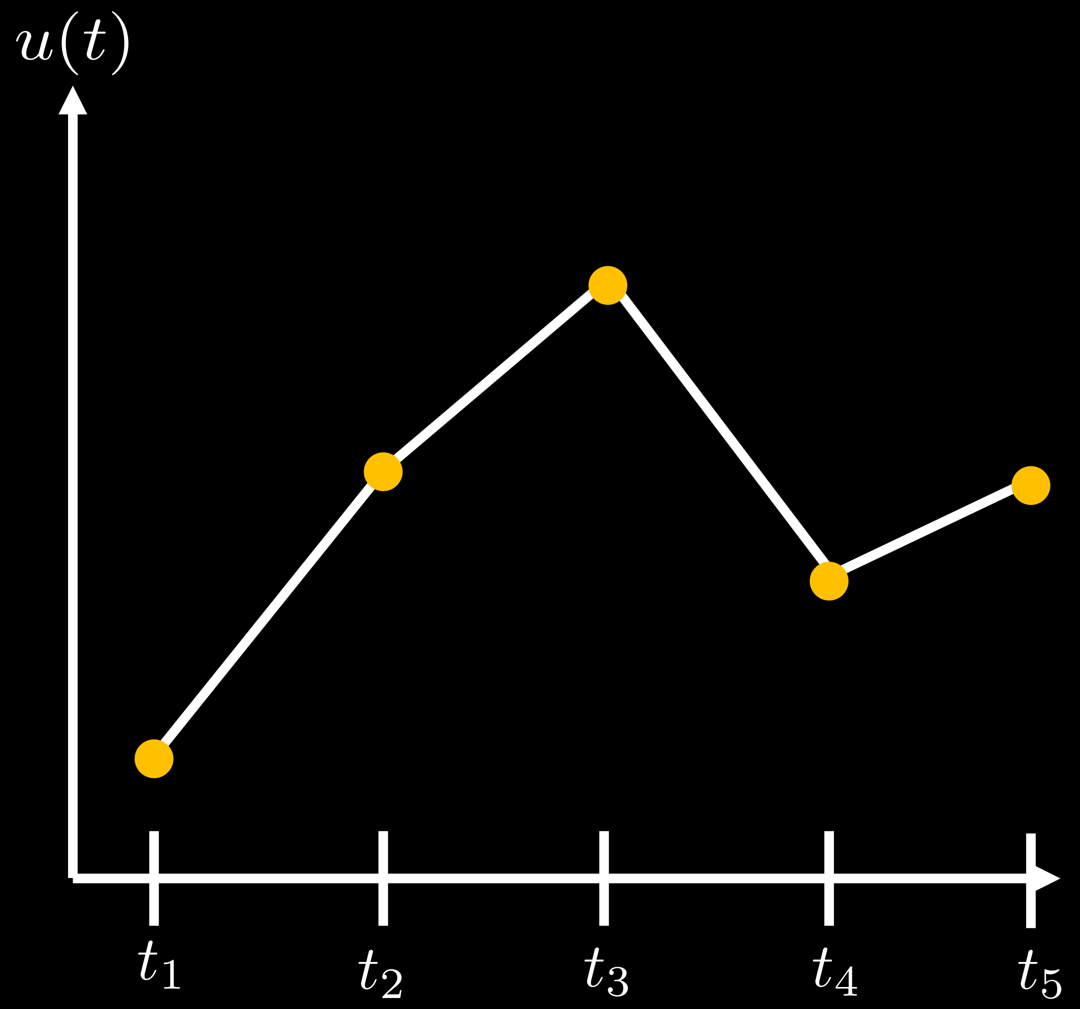
We need some way to parameterize our control in continuous time.

Zero Order Hold (ZOH) keeps the control signal constant for the window.

[Elango 2025]

Preliminaries: CT-SCvx

Optimal Control Problem → Path Constraint Reformulation → Parameterization

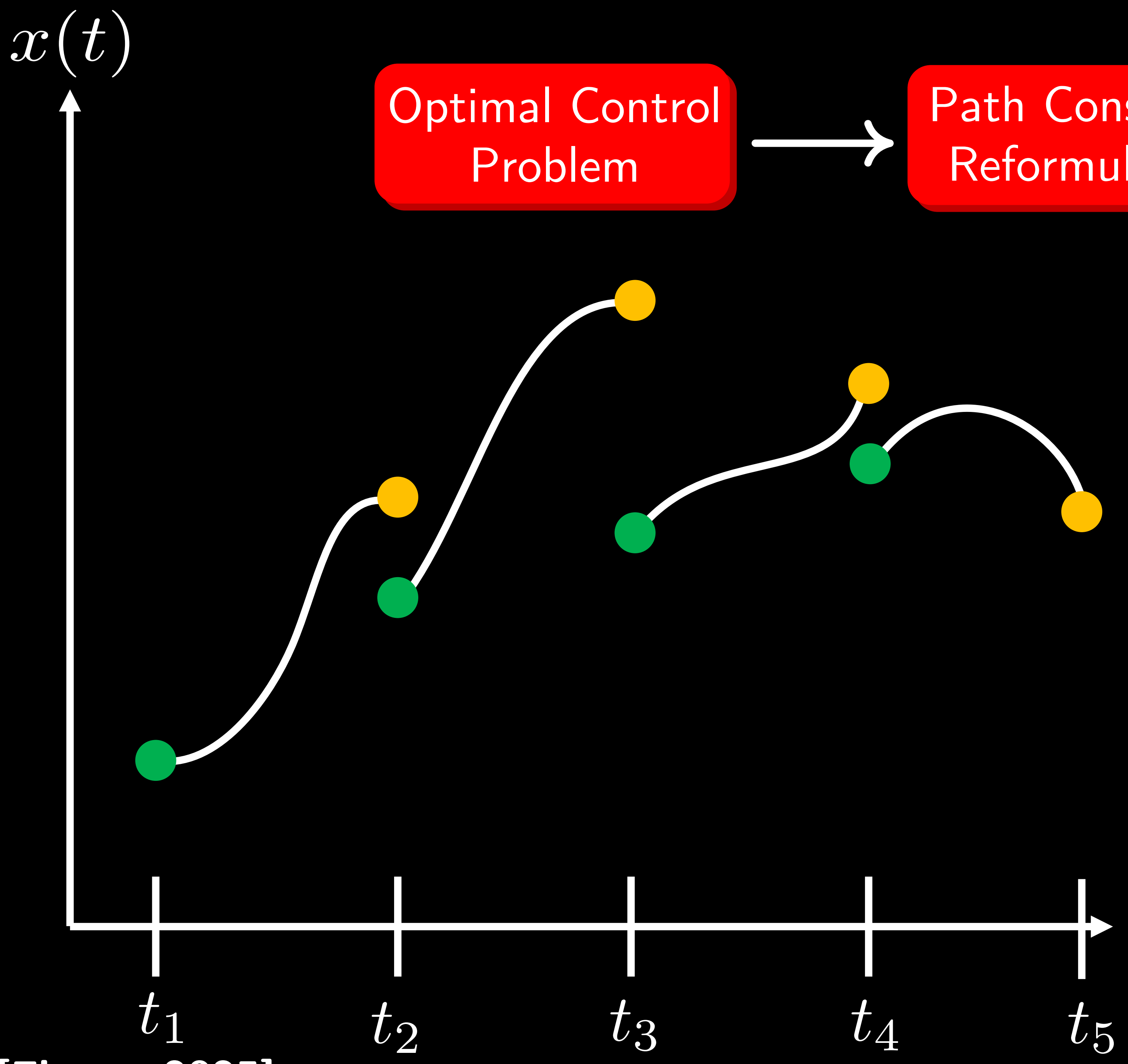


We need some way to parameterize our control in continuous time.

First Order Hold (FOH) is a piecewise linear interpretation.

[Elango 2025]

Preliminaries: CT-SCvx

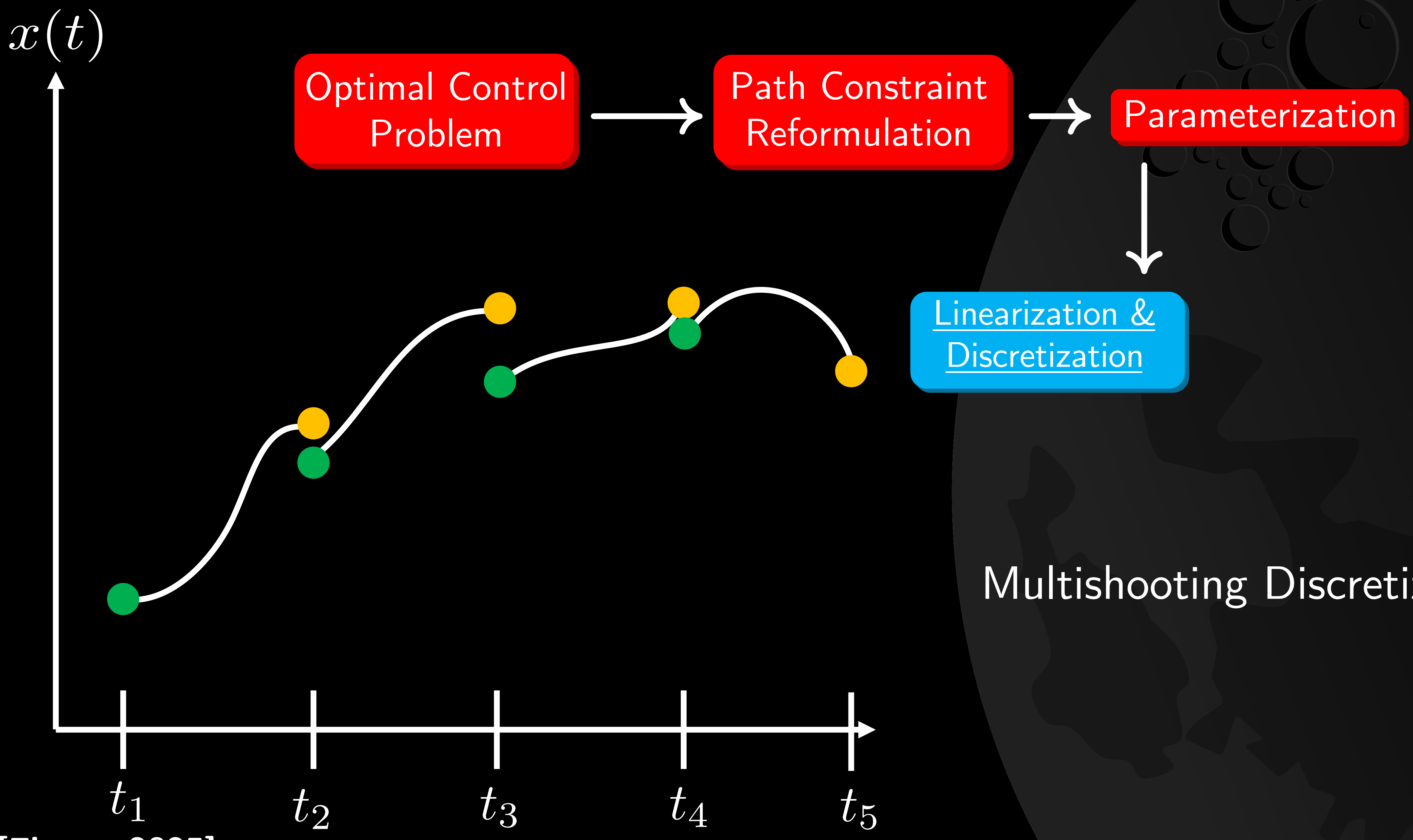


Optimal Control Problem → Path Constraint Reformulation → Parameterization → Linearization & Discretization

Multishooting Discretization

[Elango 2025]

Preliminaries: CT-SCvx



Multishooting Discretization

[Elango 2025]

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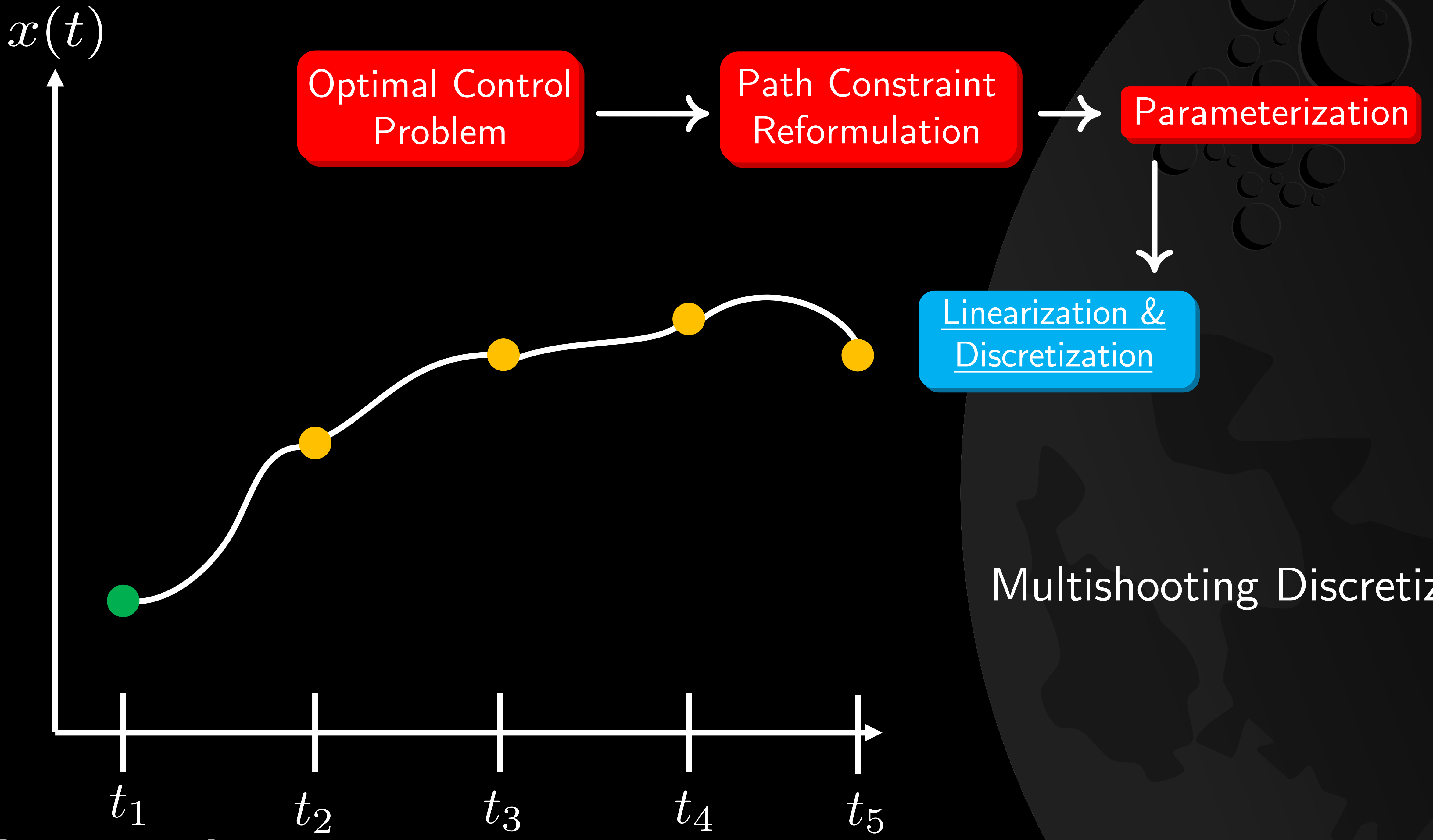
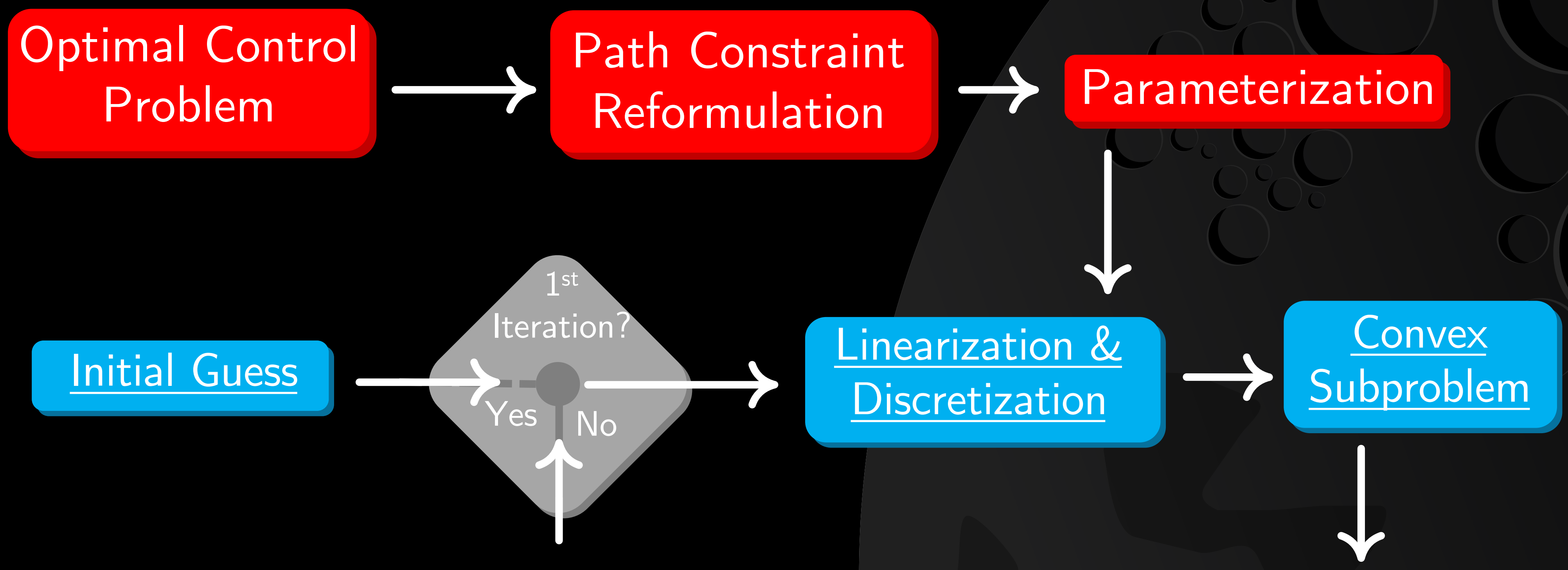


Multishooting Discretization

[Elango 2025]

Preliminaries: CT-SCvx



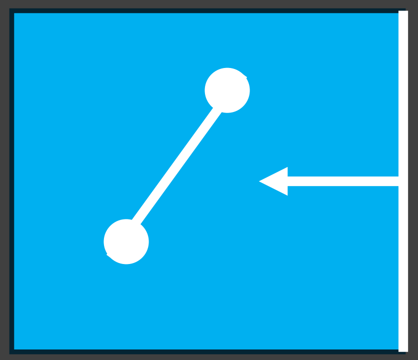
Preliminaries: CT-SCvx - Convexity

Convex

$||x||_p \leq 1$

P-Norm Ball

$Ax \leq b$



Halfspace

$||Ax - b||_2 \leq c^\top x + d$

Second Order Cone

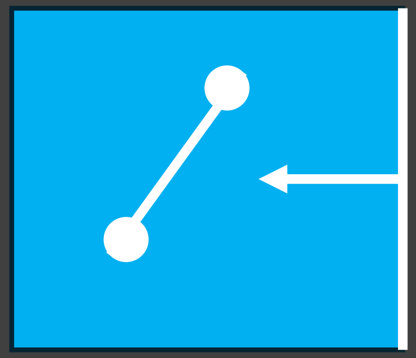
Preliminaries: CT-SCvx - Convexity

Convex

$||x||_p \leq 1$

P-Norm Ball

$Ax \leq b$

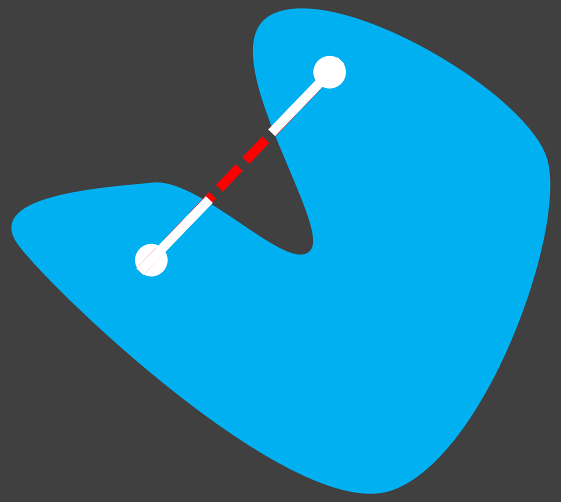


Halfspace

$||Ax - b||_2 \leq c^\top x + d$

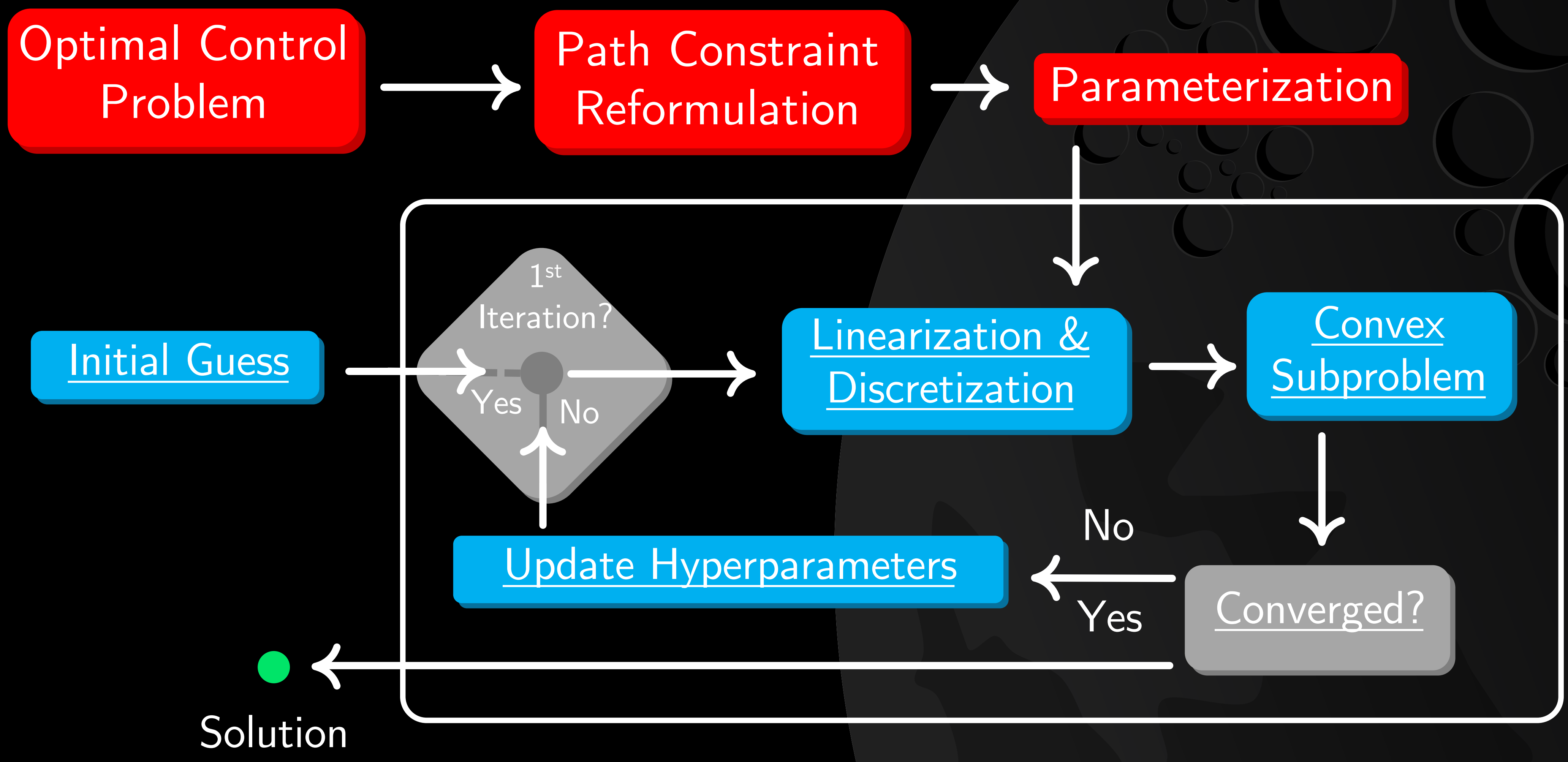
Second Order Cone

Nonconvex

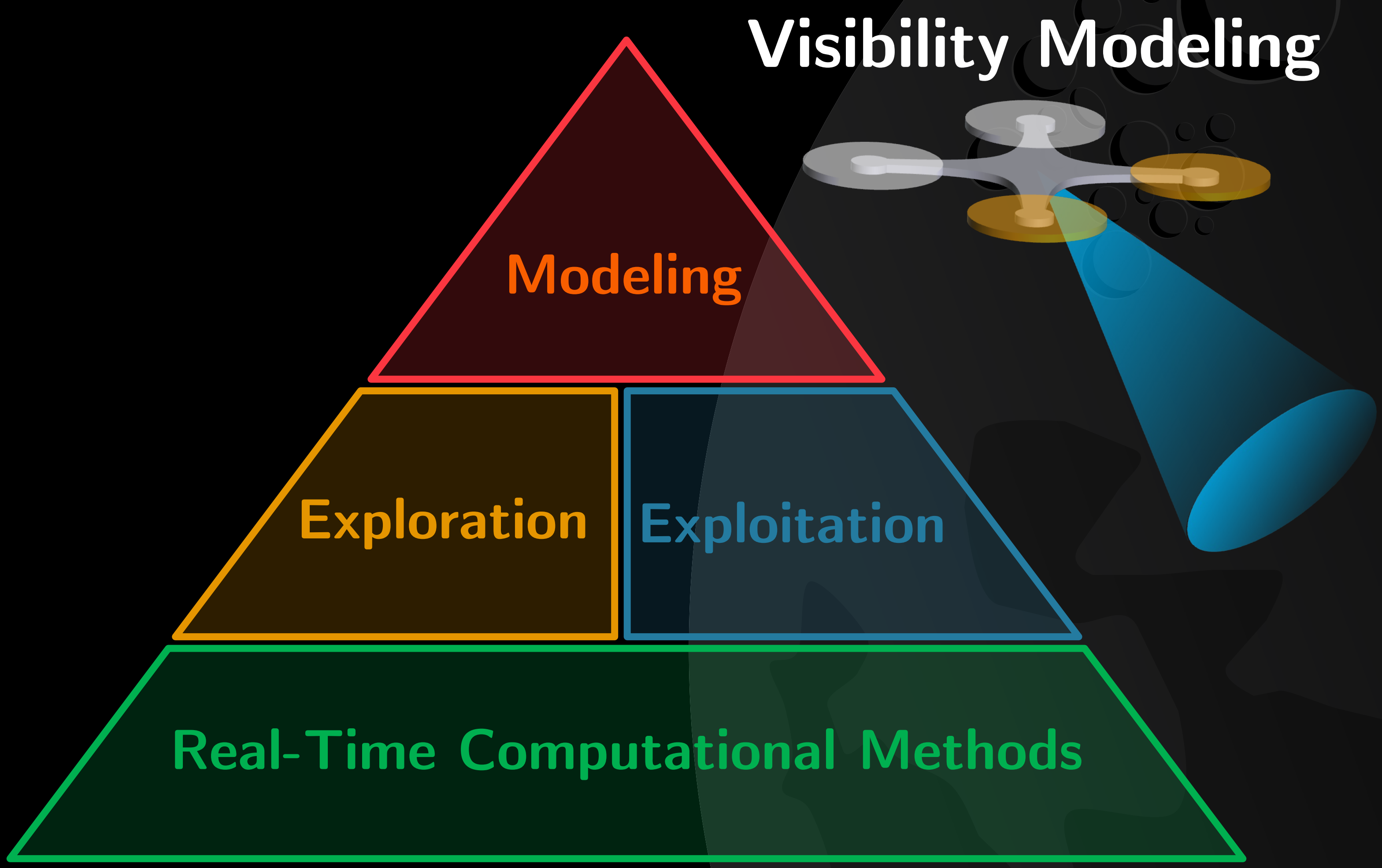


Preliminaries: CT-SCvx - Convex Optimization

Preliminaries: CT-SCvx



Roadmap: Visibility Modeling



Visibility Modeling: Goal

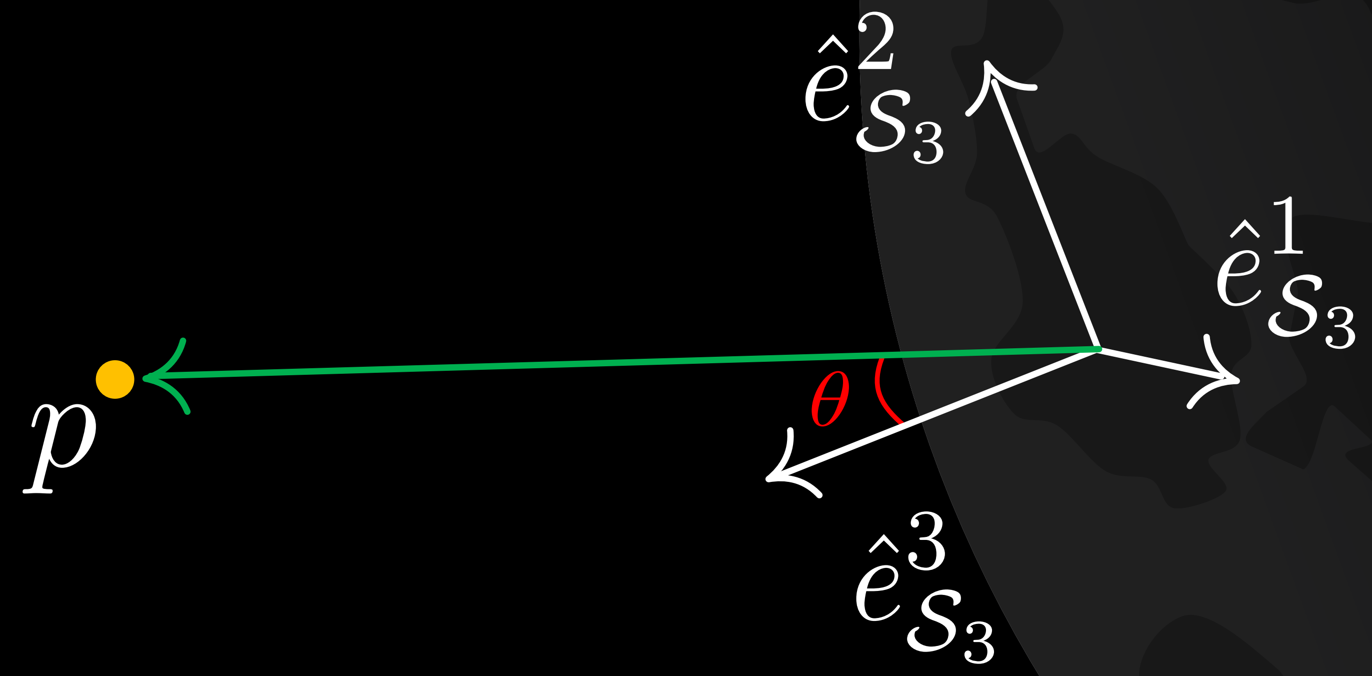
Visibility modeling aims to establish a sensor-agnostic mathematical model that determines if a spatial element is within the line-of-sight of a sensor.

Visibility Modeling: Literature

Visibility Modeling has been studied in the context of drones and other aerial platforms

Using cameras under the pinhole assumption, minimize the angle between the boresight vector of the sensor and the point to be contained within the LoS **[Papanikolopoulos 1993, Hurak 2012, Falanga 2018]**.

- ***Limitations***: Doesn't offer guarantees of containing the target point within LoS, overly conservative

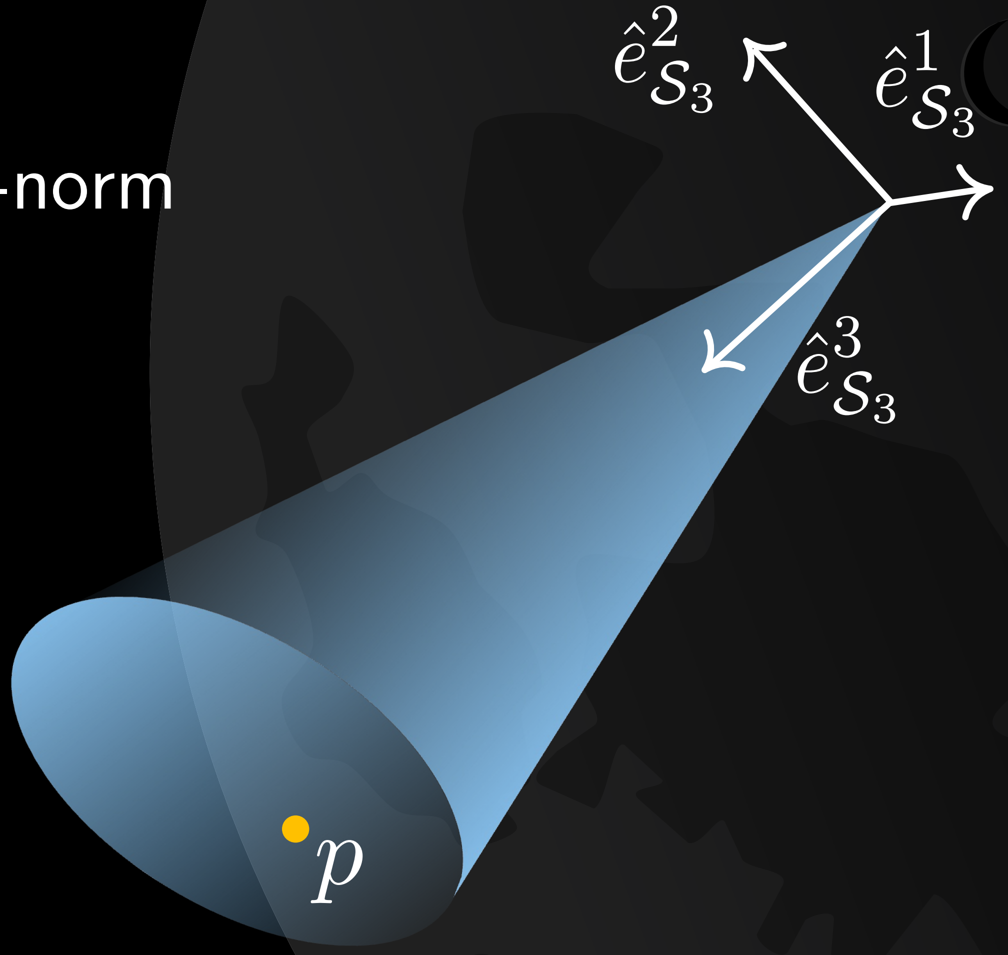


Visibility Modeling: Literature

Visibility Modeling has been studied in the context of drones and other aerial platforms

Using the dual quaternion **[Reynolds 2019]** and dot product **[Malyuta 2023, Buckner 2024]** forms of a symmetric L2 norm strictly, these works constrain a point to reside within the LoS of the view cone

- ***Limitations***: Restricted to symmetric 2-norm



Visibility Modeling

We will define a view cone, $\mathcal{K}$, as the region of Euclidean space that is visible to a sensor.

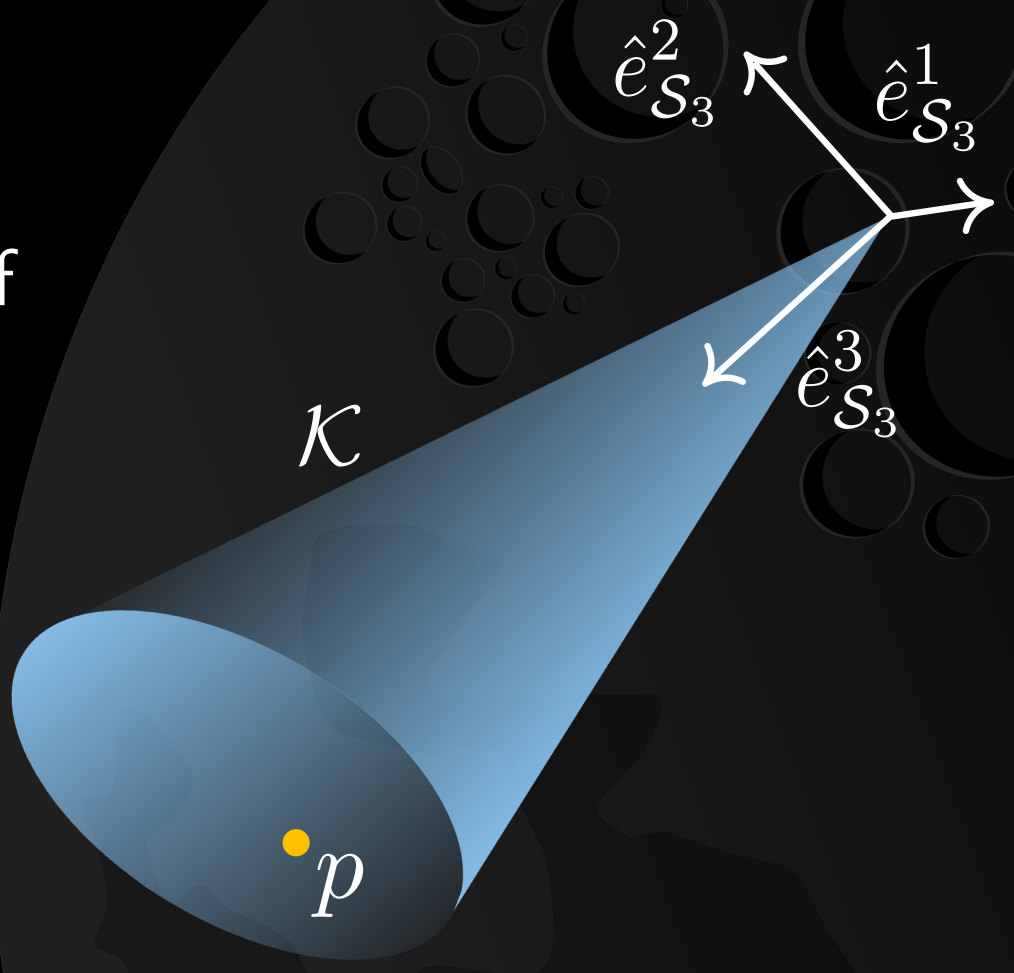


Figure: 3-Dimensional View Cone

[Hayner 2025]

Visibility Modeling: Norm Cone

Mathematically we can express the cone, $\mathcal{K}$, as follows,

Definition: N-Dimensional Norm Cone

$$\mathcal{K}(\alpha) = \{(p_{\mathcal{S}_N}^{1:N-1}, p_{\mathcal{S}_N}^{N}) \,|\, \|\mathbf{diag}(\alpha) p_{\mathcal{S}_N}^{1:N-1}\| \le p_{\mathcal{S}_N}^{N}\}$$

where,

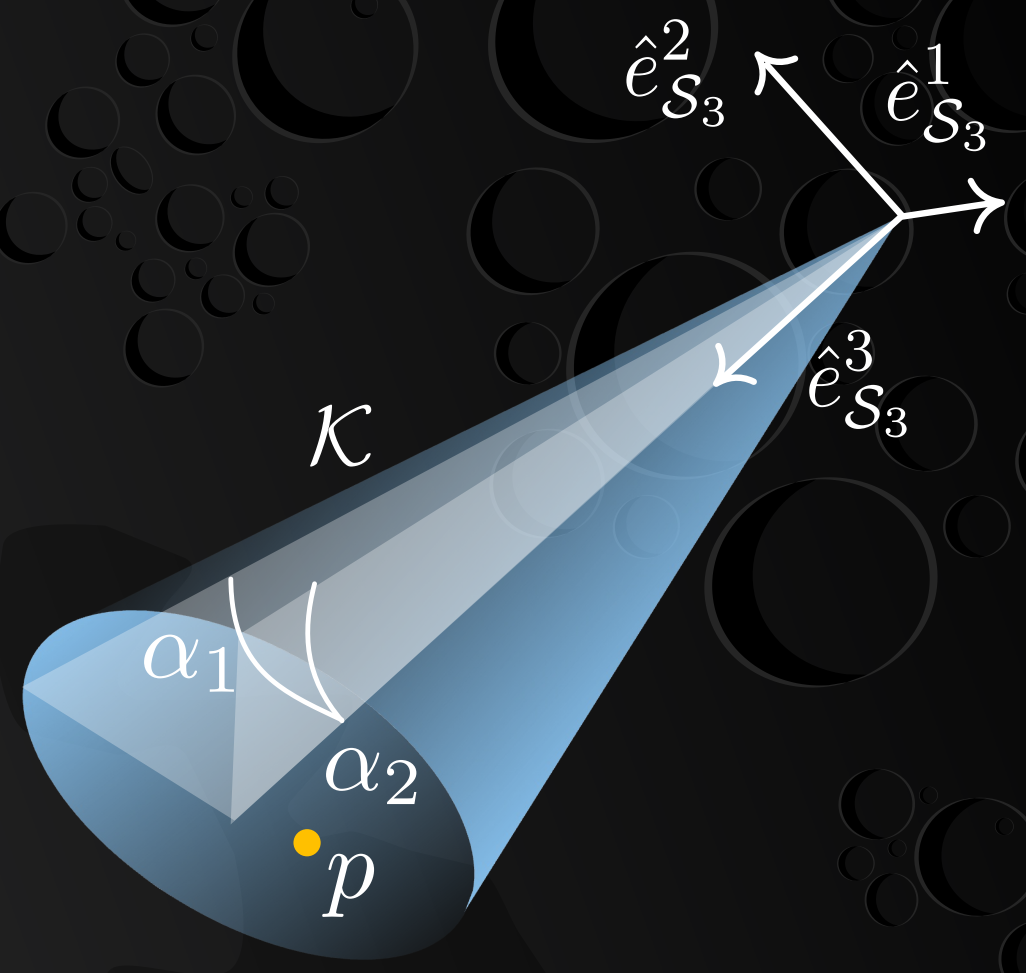


Figure: 3-Dimensional Second-Order Cone

[Boyd, 2004]

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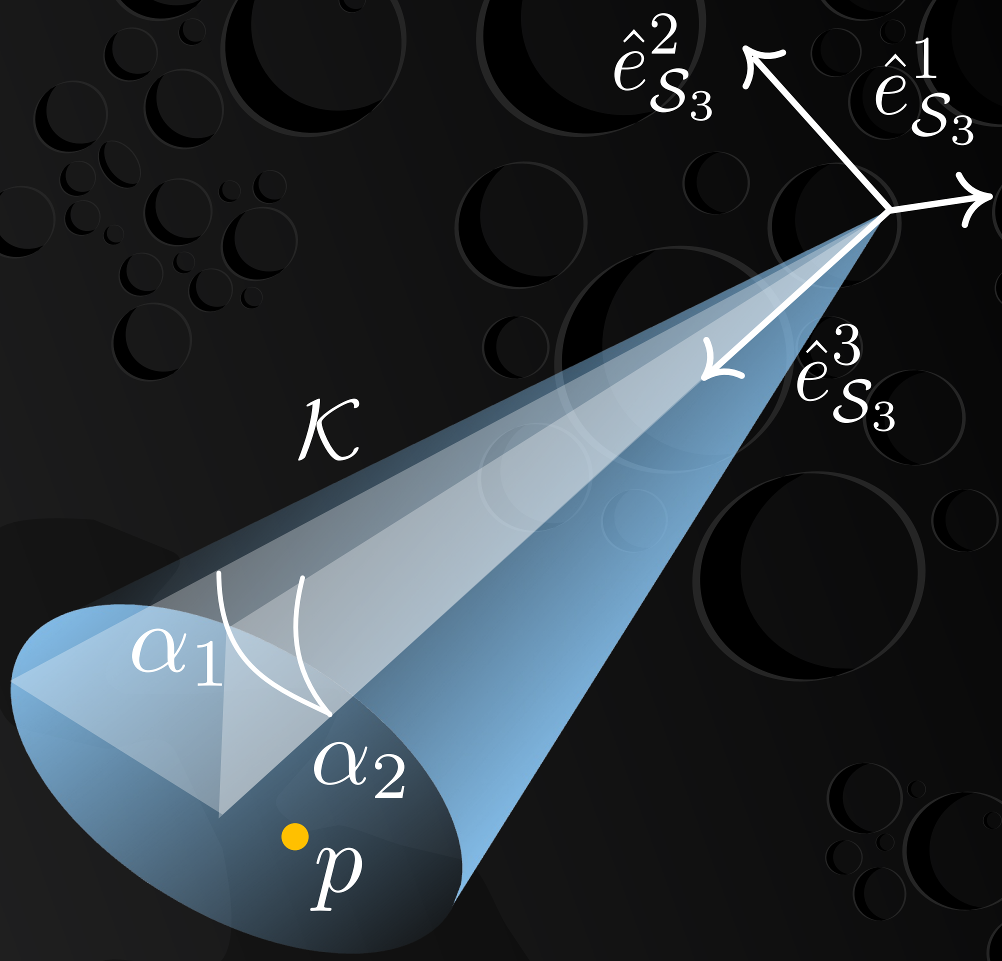
where,

$p = (p^{1:N-1}, p^{N}) \in \mathbb{R}^N$, a point in Euclidean space contained within $\mathcal{K}(\alpha)$



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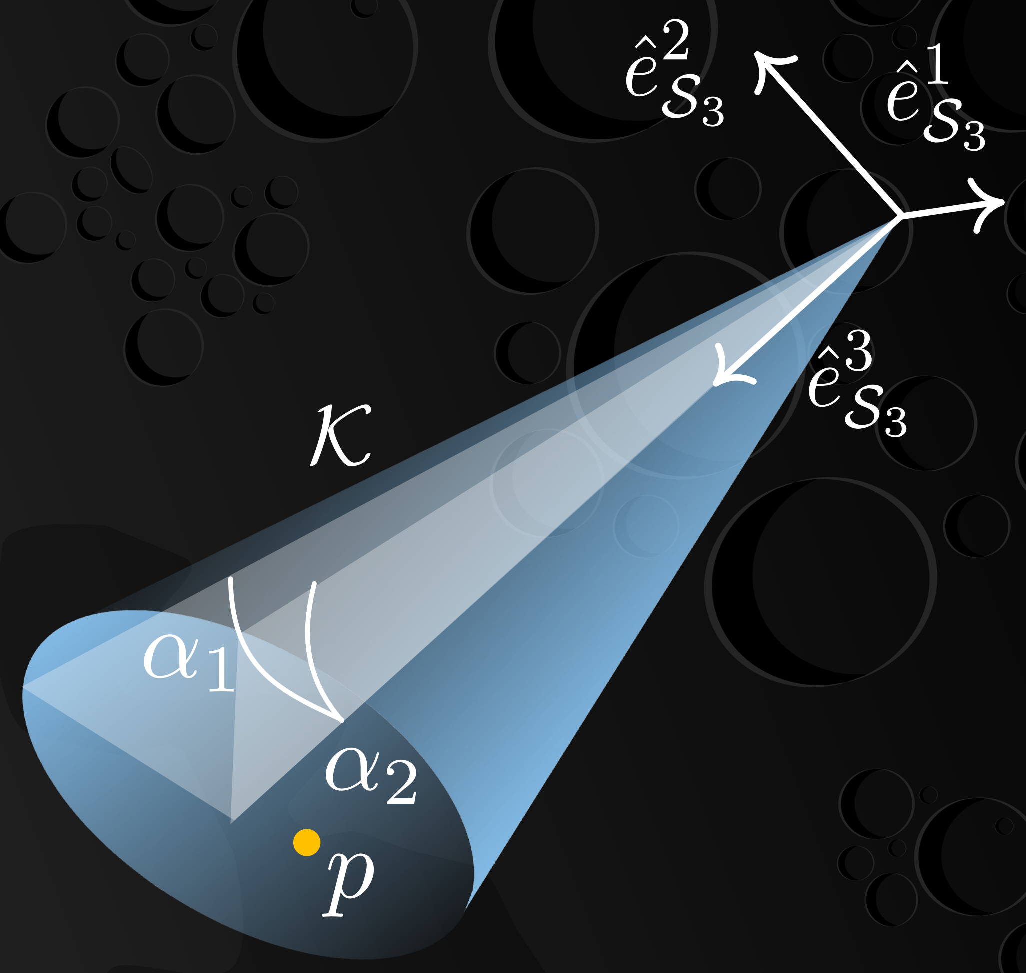
$\alpha \in \mathbb{R}^{N-1}$, each element of α is the angle of the cone in the corresponding lateral direction



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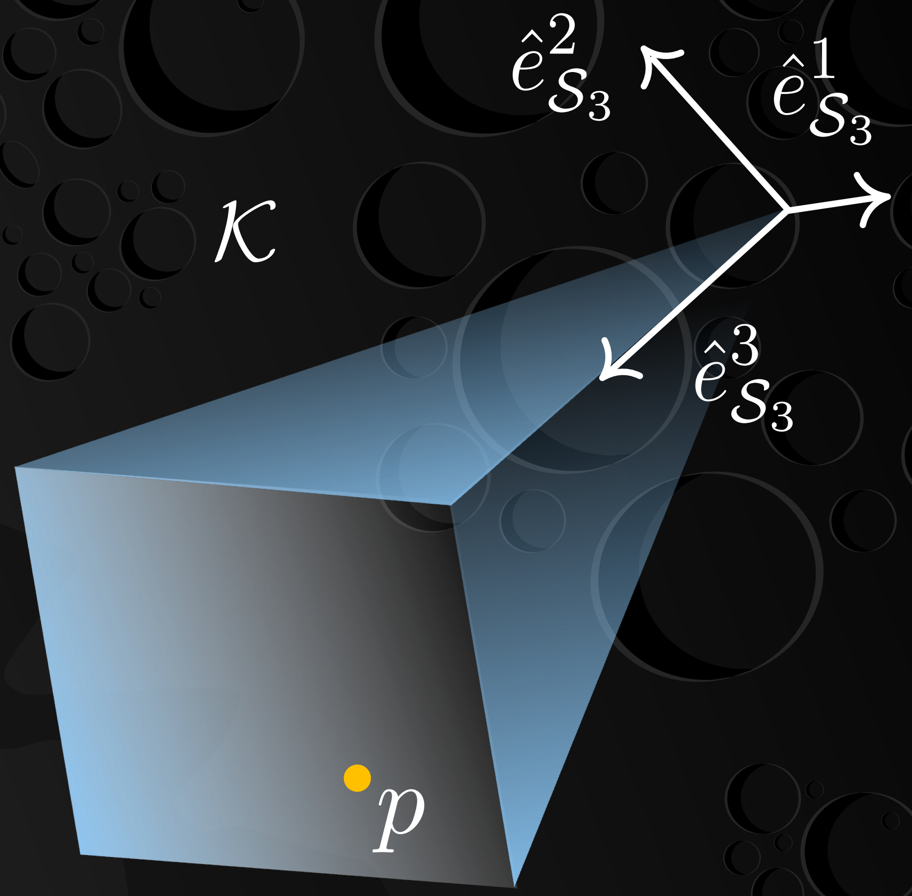

Figure: 3-Dimensional Infinity-Order Cone

[Hayner 2025]

Visibility Modeling: Transformation

Since points of interest are defined in the inertial frame, they must be resolved in the sensor frame to apply the visibility model.

Definition: Inertial to Sensor Transformation

$$p_{\mathcal{S}_N} = R_{\mathcal{I}_N \to \mathcal{S}_N}(p_{\mathcal{I}_N} - r_{\mathcal{I}_N})$$

where,

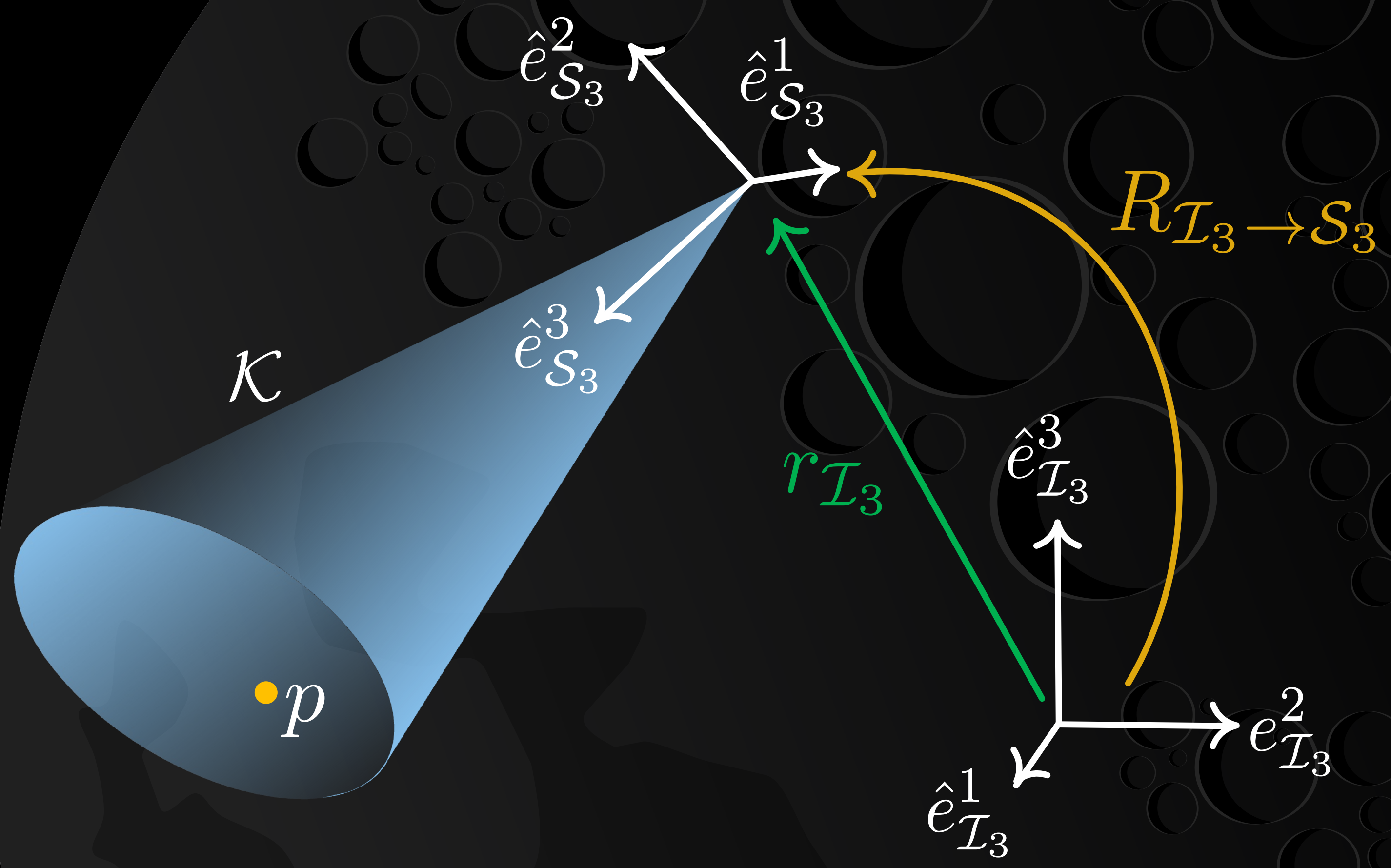

Figure: 3-Dimensional Inertial to Sensor Transformation

[Hayner 2025]

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Since points of interest are defined in the inertial frame, they must be resolved in the sensor frame to apply the visibility model.

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$$p_{\mathcal{S}_N} = R_{\mathcal{I}_N \to \mathcal{S}_N}(p_{\mathcal{I}_N} - r_{\mathcal{I}_N})$$

where,

$r_{\mathcal{I}_N} \in \mathbb{R}^N$, the position of the sensor in the inertial frame

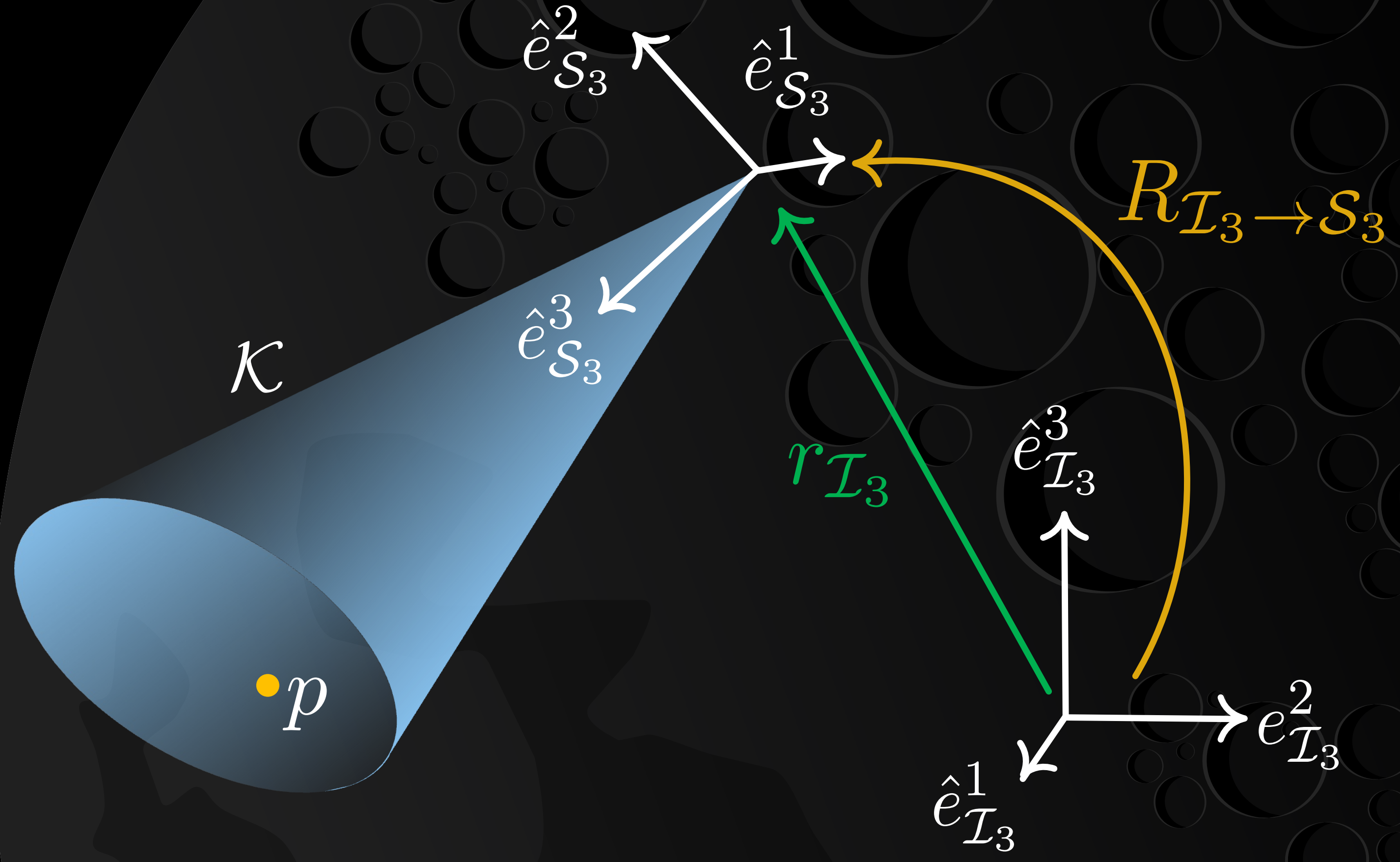

Figure: 3-Dimensional Inertial to Sensor Transformation

[Hayner 2025]

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where,

$r_{\mathcal{I}_N} \in \mathbb{R}^N$, the position of the sensor in the inertial frame

$R_{\mathcal{I}_N \to \mathcal{S}_N} \in SO(N)$, the attitude of the sensor frame to the inertial frame

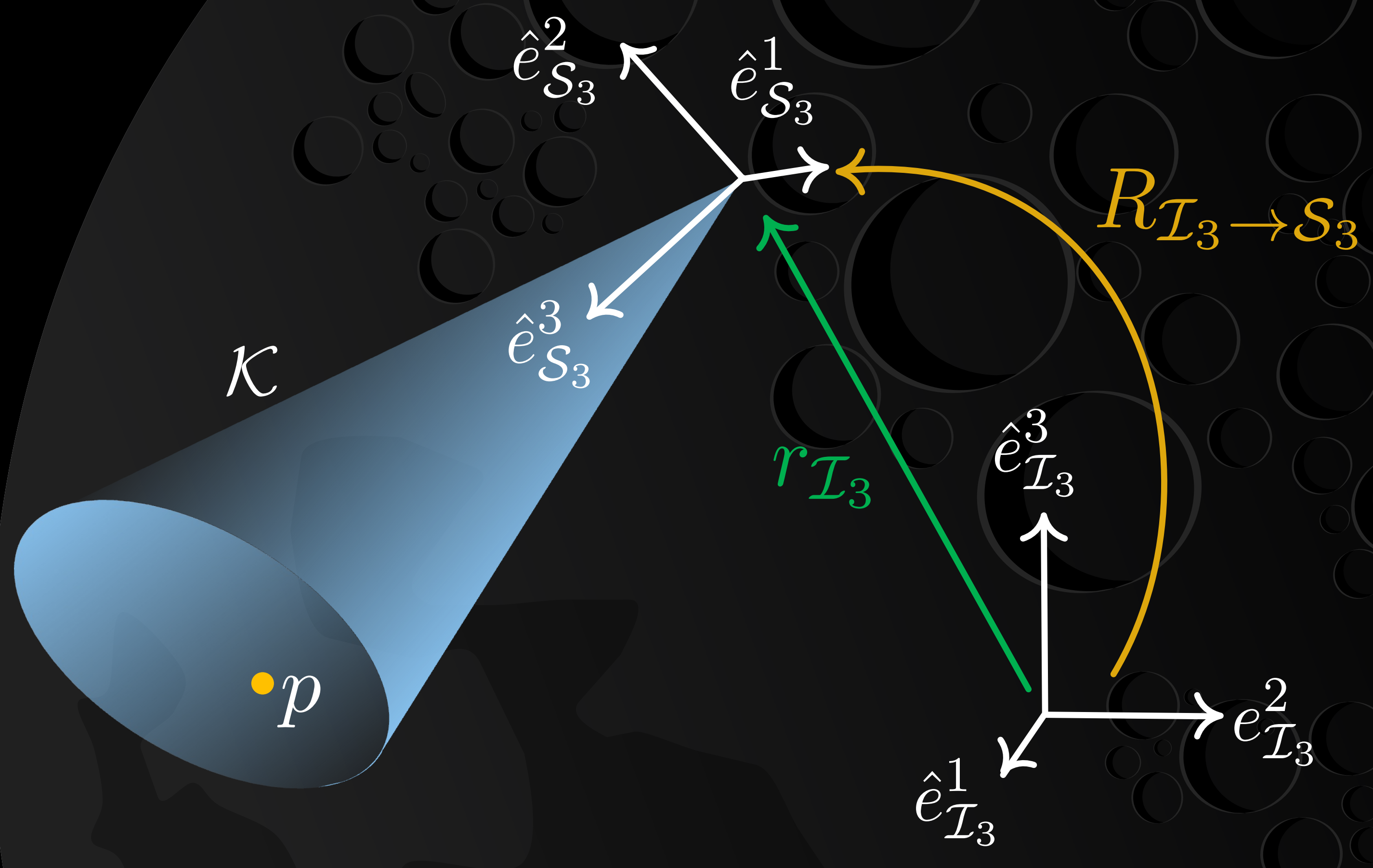


Figure: 3-Dimensional Inertial to Sensor Transformation

[Hayner 2025]

Visibility Modeling: Full Model

Definition: Full Line-of-Sight Constraint

$$g_{\mathrm{LoS}} \triangleq \|\mathbf{diag}(\alpha)[R_{\mathcal{I}_N \to \mathcal{S}_N}(p_{\mathcal{I}_N} - r_{\mathcal{I}_N})]^{1:N-1}\| - [R_{\mathcal{I}_N \to \mathcal{S}_N}(p_{\mathcal{I}_N} - r_{\mathcal{I}_N})]^{N} \leq 0$$

When the above constraint is satisfied, a point p, is in LoS

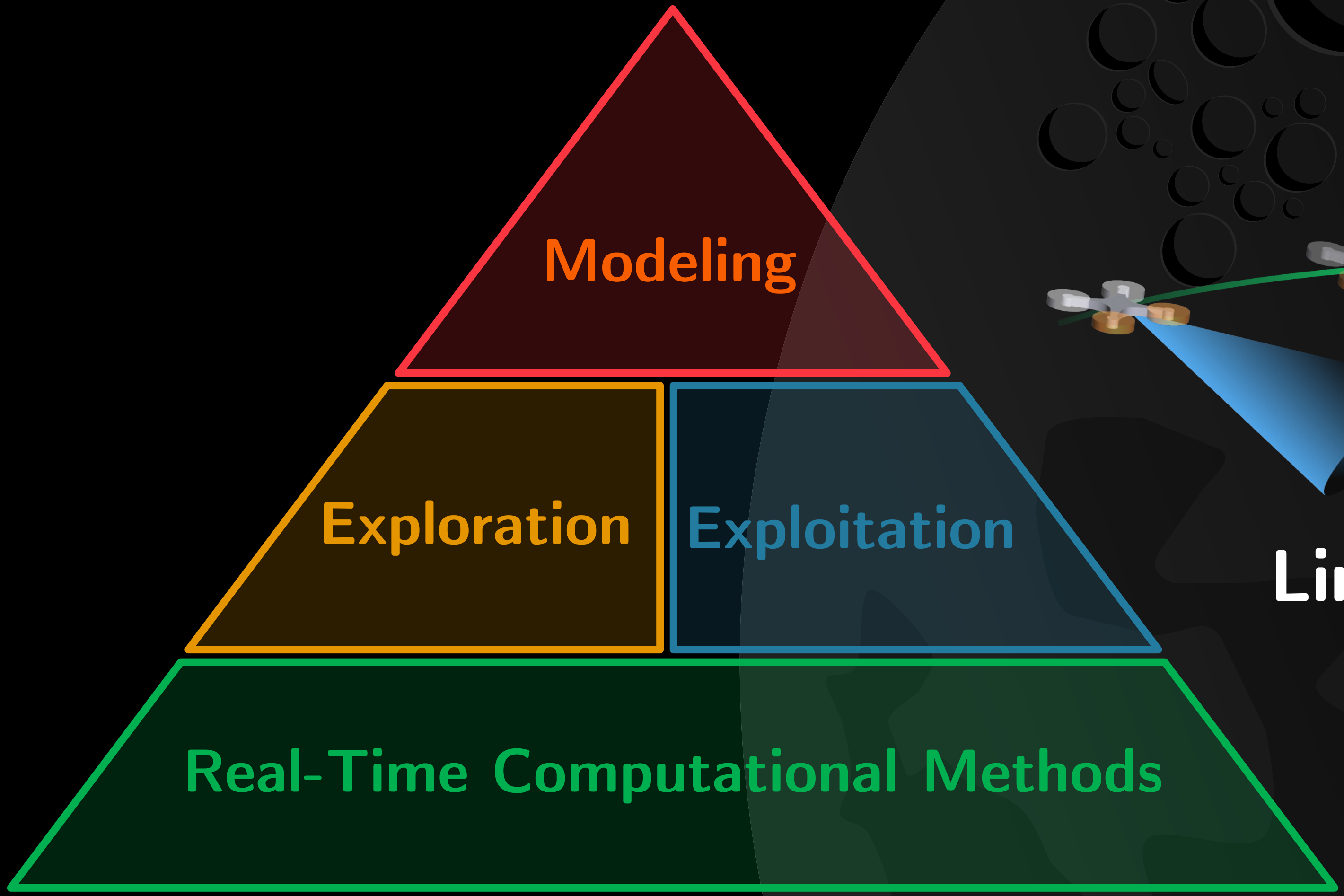
[Hayner 2025]

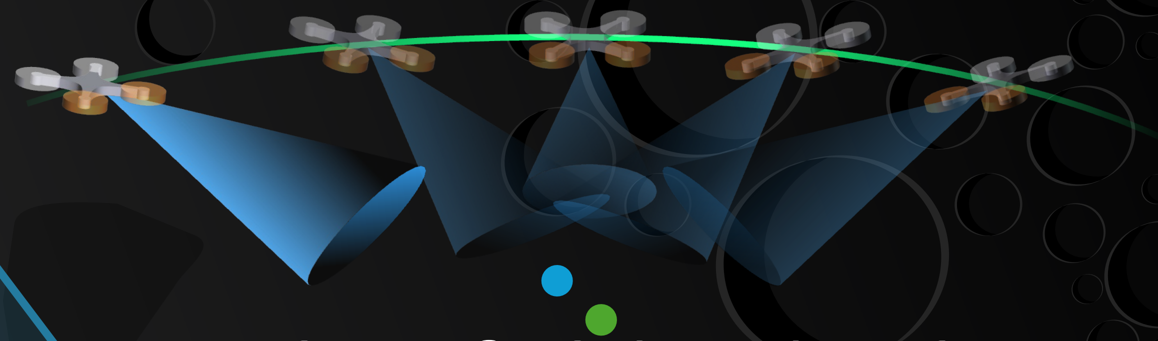
Visibility Modeling: Primary Takeaway

Key Contribution: This model parameterizes the norm type and lateral FOV angles to fit sensor requirements, allowing for a single unified visibility model to be used for cameras, LiDARs, and other exteroceptive sensors.

[Hayner 2025]

Roadmap: Exploitation





Line-of-Sight Planning

Exploitation: Goal

Exploitation methods aim to leverage specific *a priori* information within an environment to achieve a goal.

Exploitation: Literature

Trajectory planning under LoS constraints has been explored extensively,

[Mellinger 2011] introduces the differentially flat form for quadrotors, parameterized by position and a yaw or heading angle. **[Murali 2019, Spasojevic 2020]** leverage this heavily for its speed, and optimize first for a position trajectory and then a yaw angle trajectory.

- ***Limitation:*** The decoupled position *then* yaw scheme leads to sub-optimal results as the position trajectory is blind to the needs of perception.

Exploitation: Literature

Trajectory planning under LoS constraints has been explored extensively,

[Zhou 2021, Tordesillas 2022] introduces a coupled position/yaw methods which still leverage the differentially flat quadrotor model. **[Penin 2018]** further leverages differential flatness and ensures the vehicle maintains LoS using a symmetric 2-norm and avoids occlusion from spherical obstacles.

- **Limitations:** Restricted to differentially flat systems and symmetric 2-norm cones

Exploitation: Drone Racing under LoS

Drone racing with relative navigation presents a challenge where *continuous* landmark visibility is critical for state estimation, necessitating the use of the visibility model.

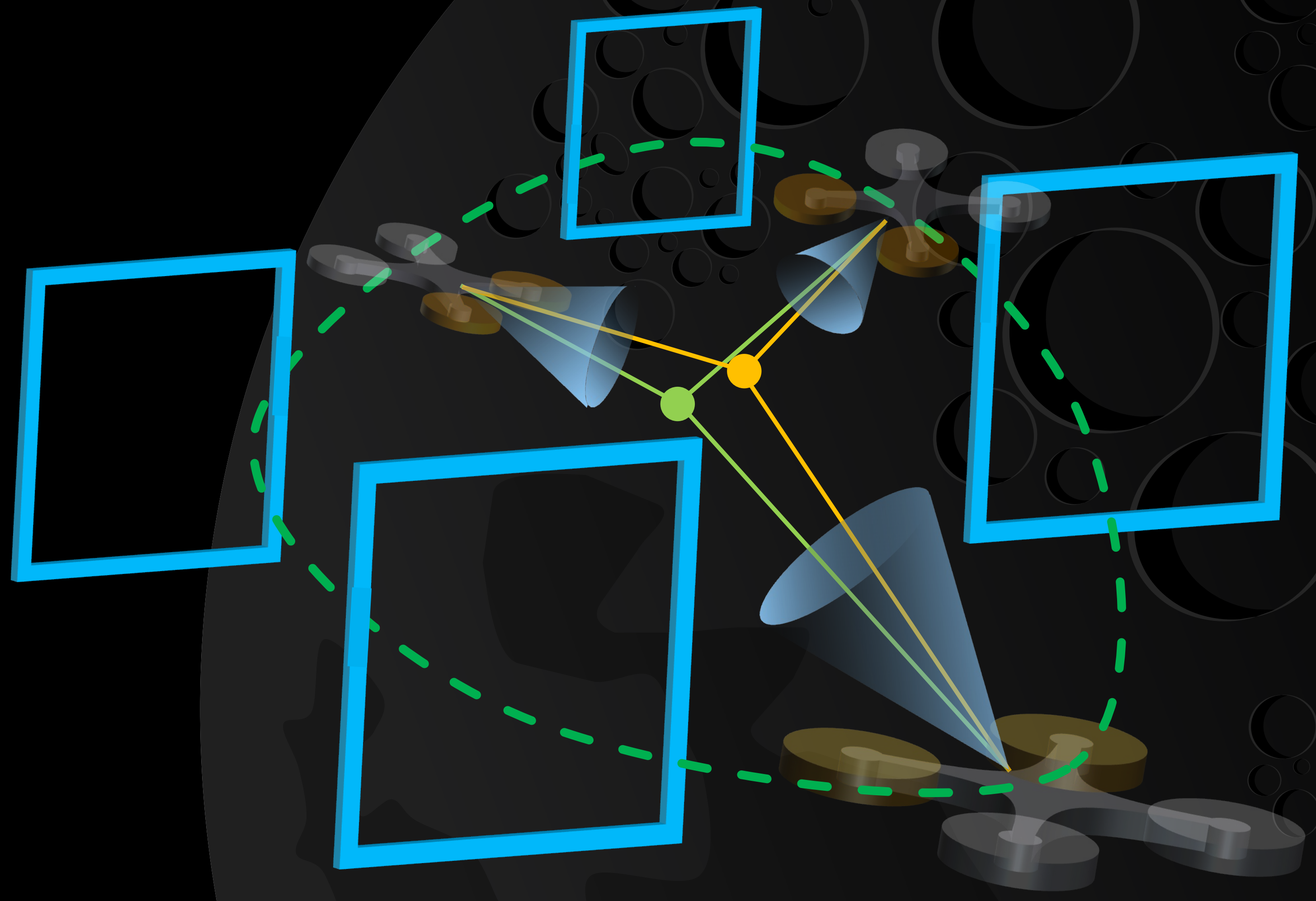


Figure: Drone Racing under LoS Constraints

Exploitation: 6 DoF Rigid Body Dynamics

[Szmuk 2019]

State

$$x = \left[r_{\mathcal{I}_3}^\top \in \mathbb{R}^3 \quad v_{\mathcal{I}_3}^\top \in \mathbb{R}^3 \quad q_{\mathcal{B}_3 \to \mathcal{I}_3}^\top \in \mathcal{S}^3 \quad \omega_{\mathcal{B}_3}^\top \in \mathbb{R}^3 \right]^\top \in \mathbb{R}^{13}$$

Exploitation: 6 DoF Rigid Body Dynamics

[Szmuk 2019]

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$$x = \left[r_{\mathcal{I}_3}^\top \in \mathbb{R}^3 \quad v_{\mathcal{I}_3}^\top \in \mathbb{R}^3 \quad q_{\mathcal{B}_3 \to \mathcal{I}_3}^\top \in \mathcal{S}^3 \quad \omega_{\mathcal{B}_3}^\top \in \mathbb{R}^3 \right]^\top \in \mathbb{R}^{13}$$

Control $u = \left[T_{\mathcal{B}_3}^\top \in \mathbb{R}^3 \quad M_{\mathcal{B}_3}^\top \in \mathbb{R}^3 \right]^\top \in \mathbb{R}^6$

Exploitation: 6 DoF Rigid Body Dynamics

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Control

$$u = \left[T_{\mathcal{B}_3}^\top \in \mathbb{R}^3 \quad M_{\mathcal{B}_3}^\top \in \mathbb{R}^3 \right]^\top \in \mathbb{R}^6$$

Dynamics

Position: $$\dot{r}_{\mathcal{I}_3}(t) = v_{\mathcal{I}_3}(t),$$

Velocity: $$\dot{v}_{\mathcal{I}_3}(t) = \frac{1}{m} C(q_{\mathcal{B}_3 \to \mathcal{I}_3}(t)) T_{\mathcal{B}_3}(t) + g_{\mathcal{I}_3},$$

Attitude: $$\dot{q}_{\mathcal{B}_3 \to \mathcal{I}_3}(t) = \frac{1}{2} \Omega(\omega_{\mathcal{B}_3}(t)) q_{\mathcal{B}_3 \to \mathcal{I}_3}(t),$$

Angular Rate: $$\dot{\omega}_{\mathcal{B}_3}(t) = J_{\mathcal{B}_3}^{-1} \left(M_{\mathcal{B}_3}(t) - [\omega_{\mathcal{B}_3}(t)]_\times J_{\mathcal{B}_3} \omega_{\mathcal{B}_3}(t) \right),$$

Drone Racing under LoS

Objective: Minimum Time
Constraints: Line-of-sight on keypoints
Gates
Boundary Constraints
Box Constraints
Viewcone: Symmetric 2-Norm cone
Number of Keypoints: 10

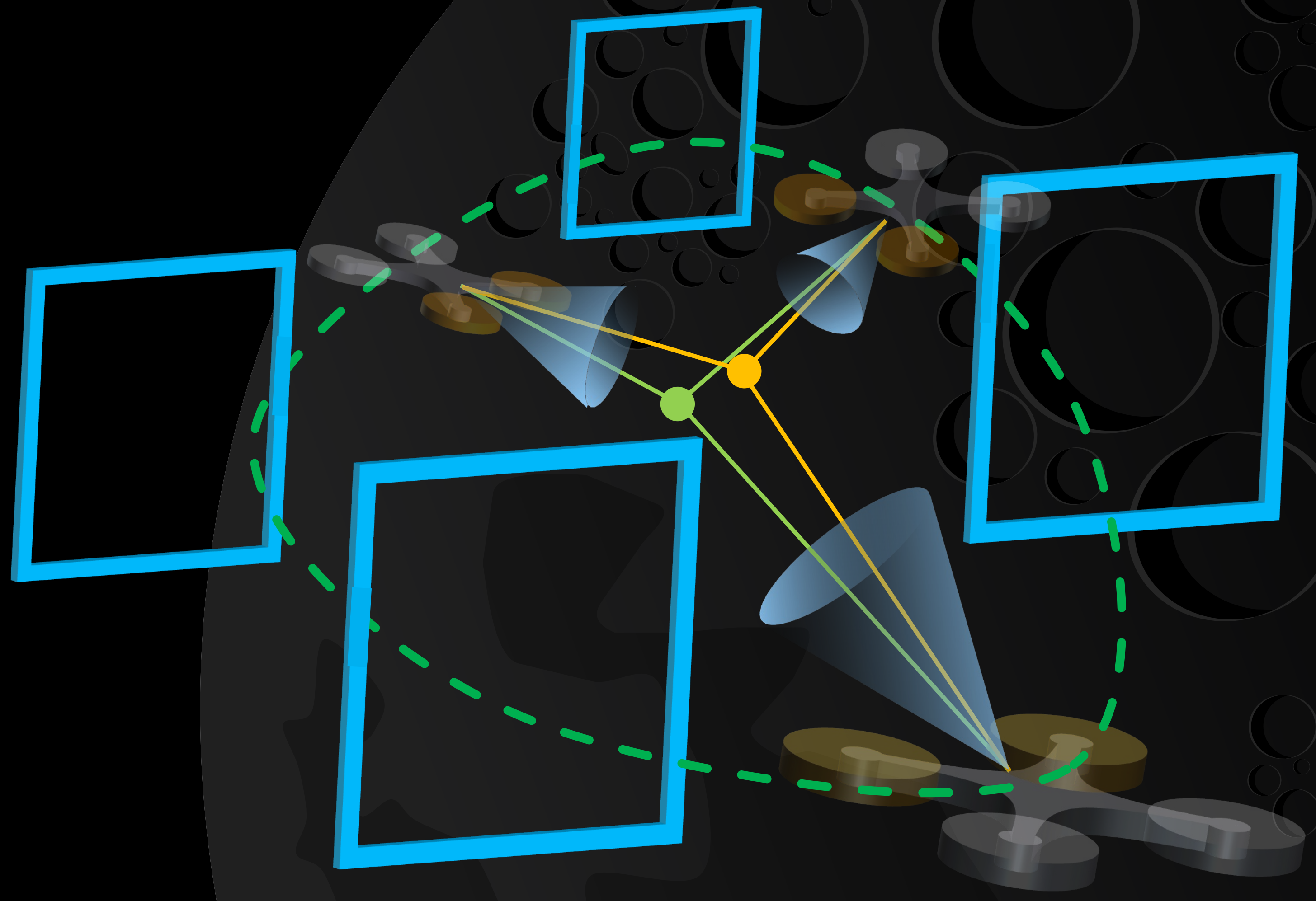

Figure: Drone Racing under LoS Constraints

Drone Racing under LoS: Problem Form

$$
\begin{aligned}
\min_{r_{\mathcal{I}_3}, v_{\mathcal{I}_3}, q_{\mathcal{B}_3 \to \mathcal{I}_3}, \omega_{\mathcal{B}_3}, T_{\mathcal{B}_3}, M_{\mathcal{B}_3}, t_f} \quad & t_f \\
\text{subject to} \quad & r_{\mathcal{I}_3}(0) = r^0, \quad v_{\mathcal{I}_3}(0) = v^0 \\
& q_{\mathcal{B}_3 \to \mathcal{I}_3}(0) = q^0, \quad \omega_{\mathcal{B}_3}(0) = \omega^0 \\
& r_{\mathcal{I}_3}(t_f) = r_f \\
& \dot{x}(t) = f_{\text{6DoF}}(t, x(t), u(t)) \\
& g_{\text{LoS}}(p^i_{\mathcal{I}_3}, r_{\mathcal{I}_3}(t), q_{\mathcal{B}_3 \to \mathcal{I}_3}(t), \alpha) \leq 0, \quad \forall i \in \{1, \ldots, N_k\} \\
& \|A^j (r_{\mathcal{I}_3}(t) - r^j_{\text{gate}, \mathcal{I}_3})\|_\infty \leq 1, \quad \forall j \in \{1, \ldots, N_g\} \\
& r_{\min} \leq r_{\mathcal{I}_3}(t) \leq r_{\max}, \quad v_{\min} \leq v_{\mathcal{I}_3}(t) \leq v_{\max} \\
& q_{\min} \leq q_{\mathcal{B}_3 \to \mathcal{I}_3}(t) \leq q_{\max}, \quad \omega_{\min} \leq \omega_{\mathcal{B}_3}(t) \leq \omega_{\max} \\
& T_{\min} \leq T_{\mathcal{B}_3}(t) \leq T_{\max}, \quad M_{\min} \leq M_{\mathcal{B}_3}(t) \leq M_{\max}
\end{aligned}
$$

where constraints without explicit time indices hold for all $t \in [0, t_f]$.

[Hayner 2025]

Drone Racing under LoS: Problem Form

$$\begin{aligned}
\min_{r_{\mathcal{I}_3}, v_{\mathcal{I}_3}, q_{\mathcal{B}_3 \to \mathcal{I}_3}, \omega_{\mathcal{B}_3}, T_{\mathcal{B}_3}, M_{\mathcal{B}_3}, t_f} \quad & t_f \\
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& r_{\min} \leq r_{\mathcal{I}_3}(t) \leq r_{\max}, \quad v_{\min} \leq v_{\mathcal{I}_3}(t) \leq v_{\max} \\
& q_{\min} \leq q_{\mathcal{B}_3 \to \mathcal{I}_3}(t) \leq q_{\max}, \quad \omega_{\min} \leq \omega_{\mathcal{B}_3}(t) \leq \omega_{\max} \\
& T_{\min} \leq T_{\mathcal{B}_3}(t) \leq T_{\max}, \quad M_{\min} \leq M_{\mathcal{B}_3}(t) \leq M_{\max}
\end{aligned}
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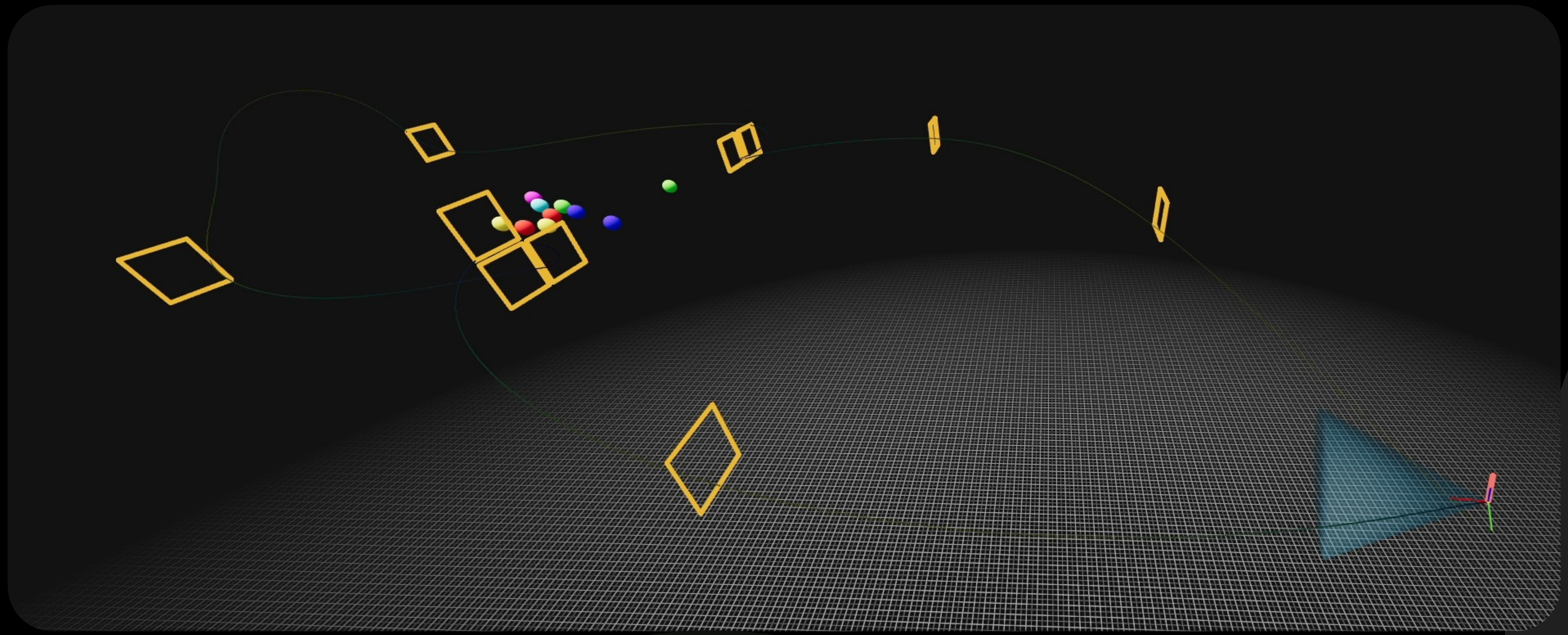
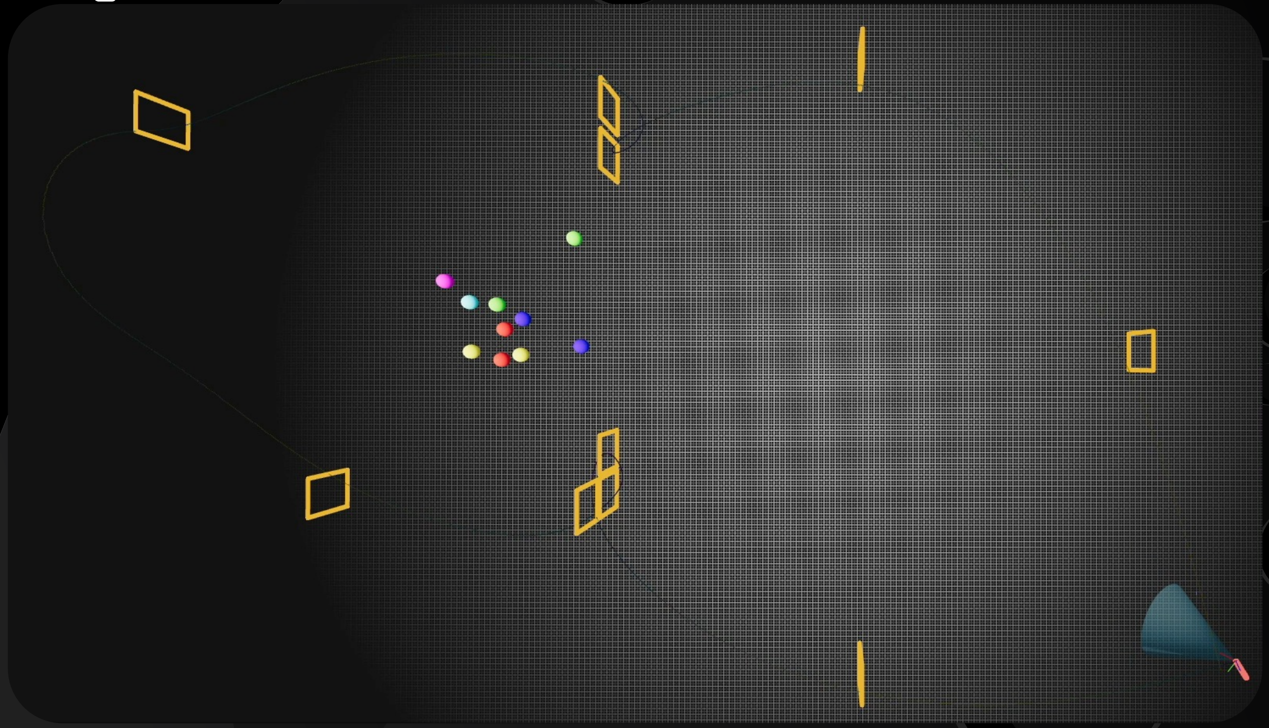
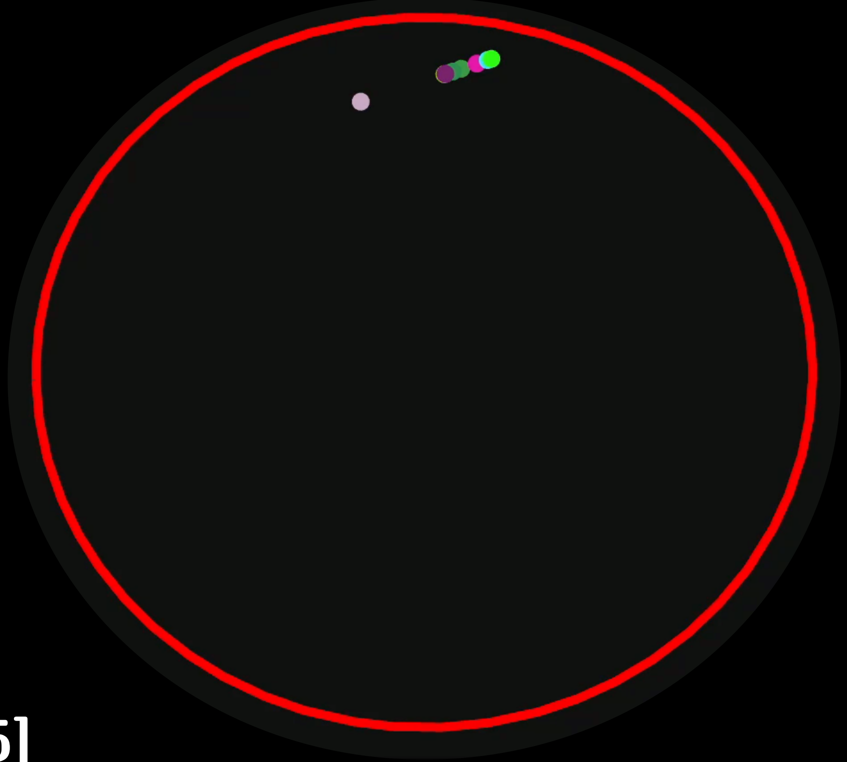
where constraints without explicit time indices hold for all $t \in [0, t_f]$.

[Hayner 2025]

Drone Racing under LoS: Qualitative Results



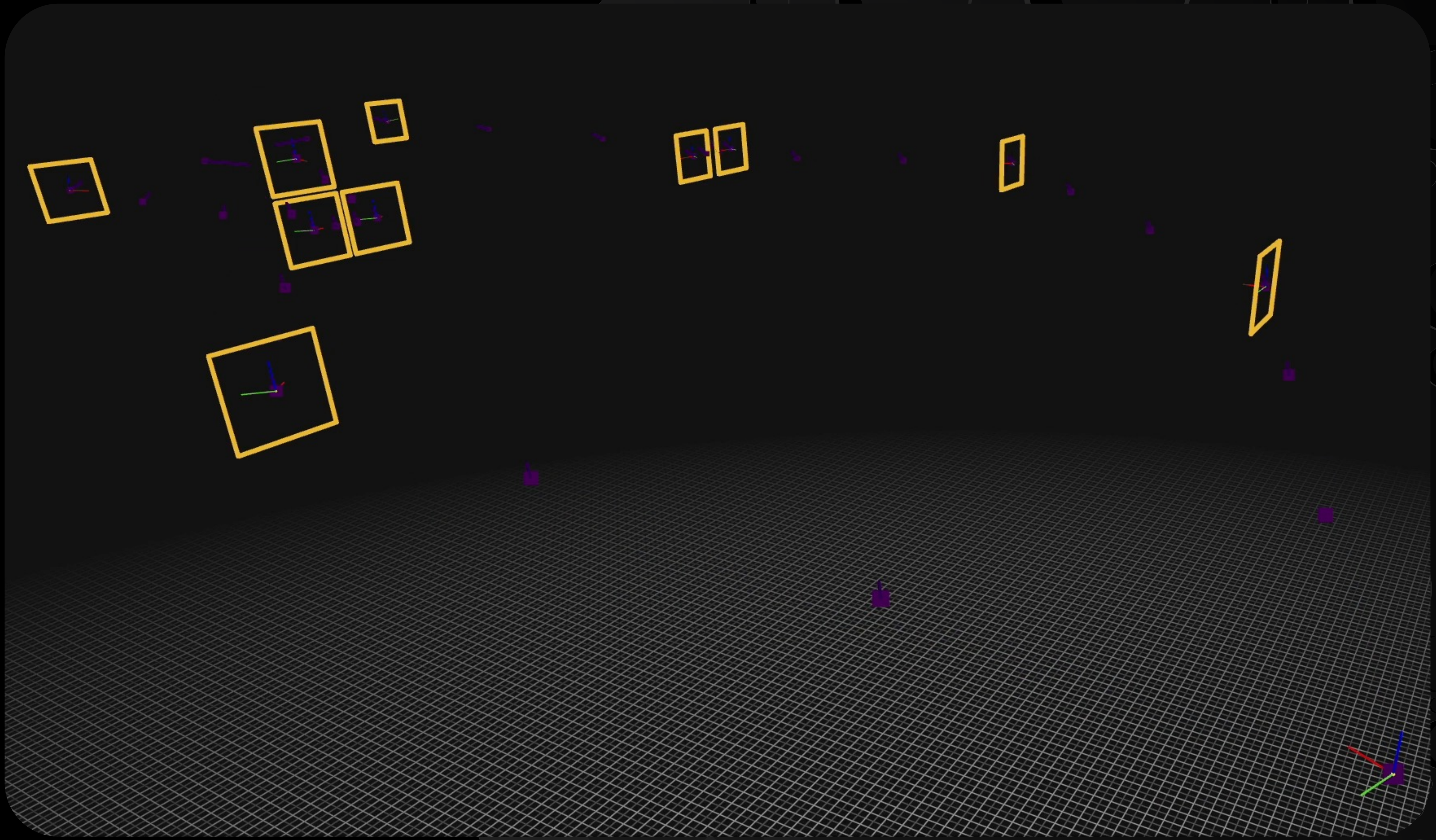




[Hayner 2025]

Drone Racing under LoS: Qualitative Results

Time of Flight: 38.78s
Iterations: 12
Solve Time: 0.491s



[Hayner 2025]

Drone Racing under LoS: Quantitative Results

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[Hayner 2025]

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[Hayner 2025]

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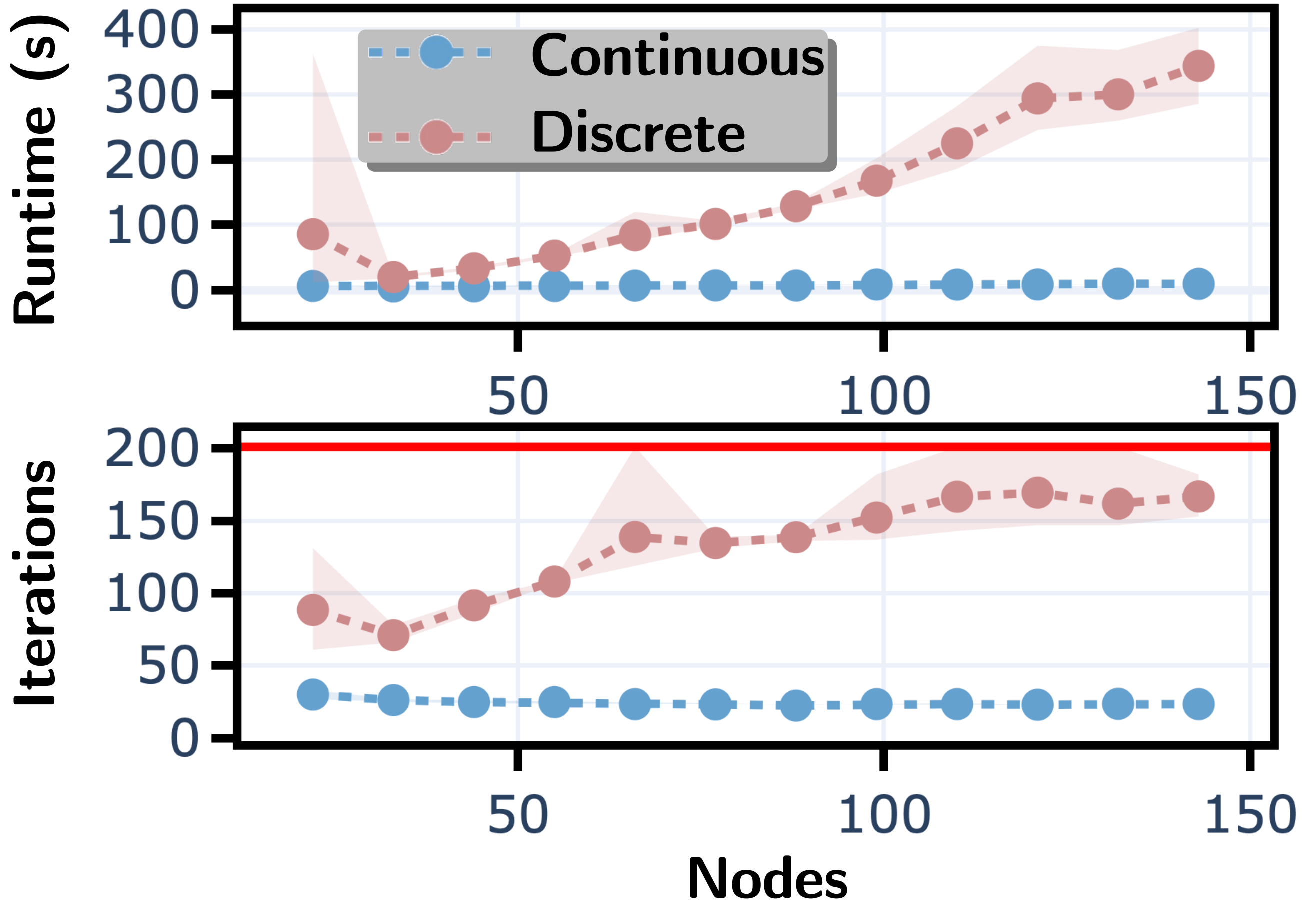
2. The total runtime
3. Original objective cost
4. The number of iterations

[Hayner 2025]

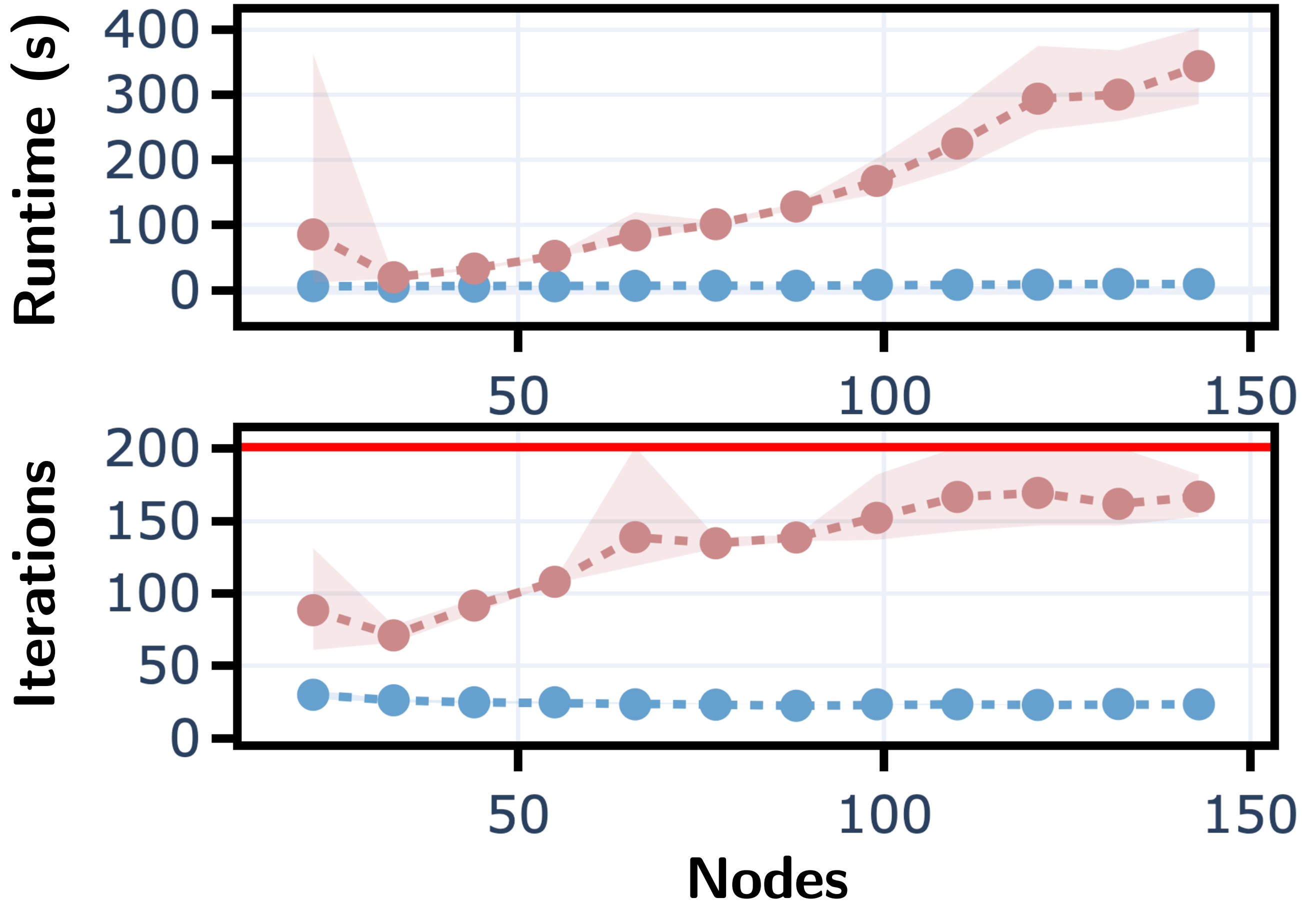
Drone Racing under LoS: Quantitative Results

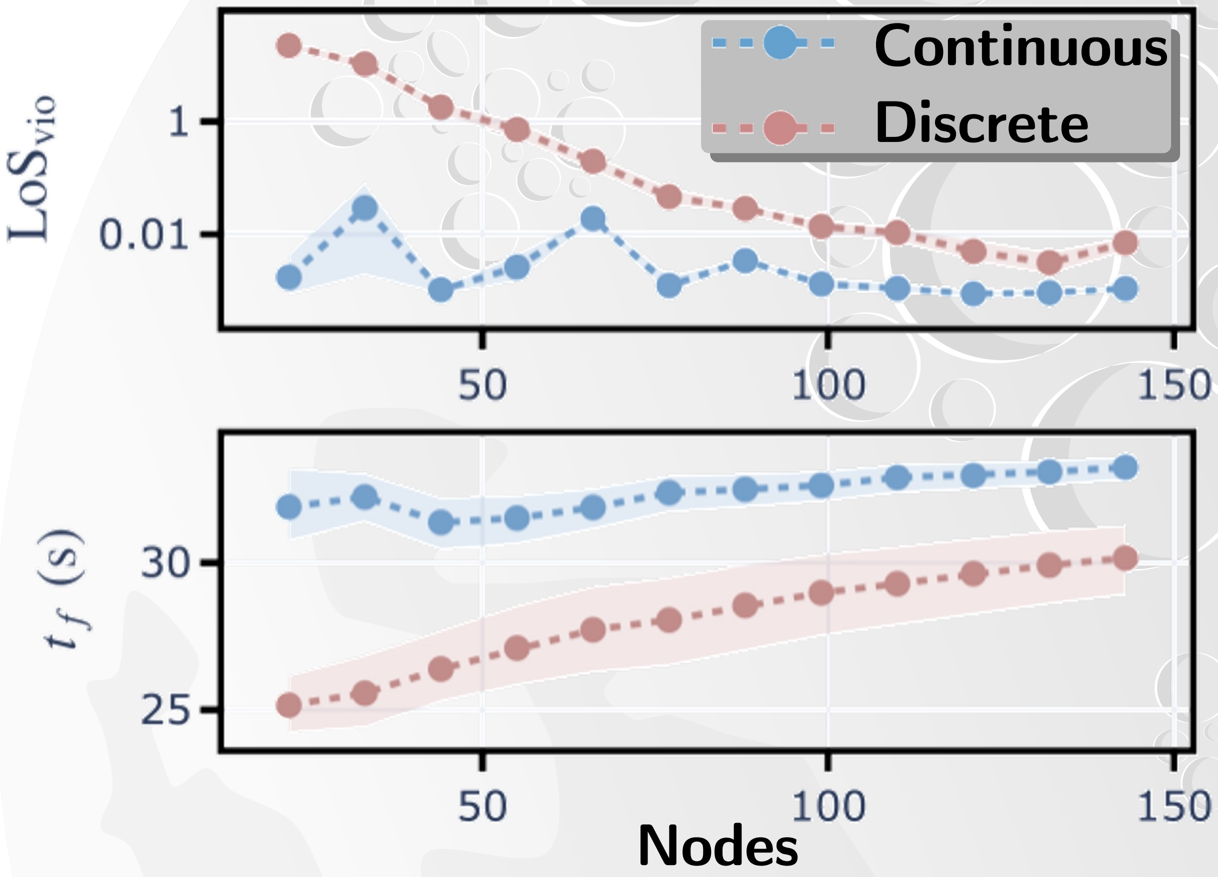


Main Takeaway

Continuous scales much better with respect to the discretization grid size

Drone Racing under LoS: Quantitative Results





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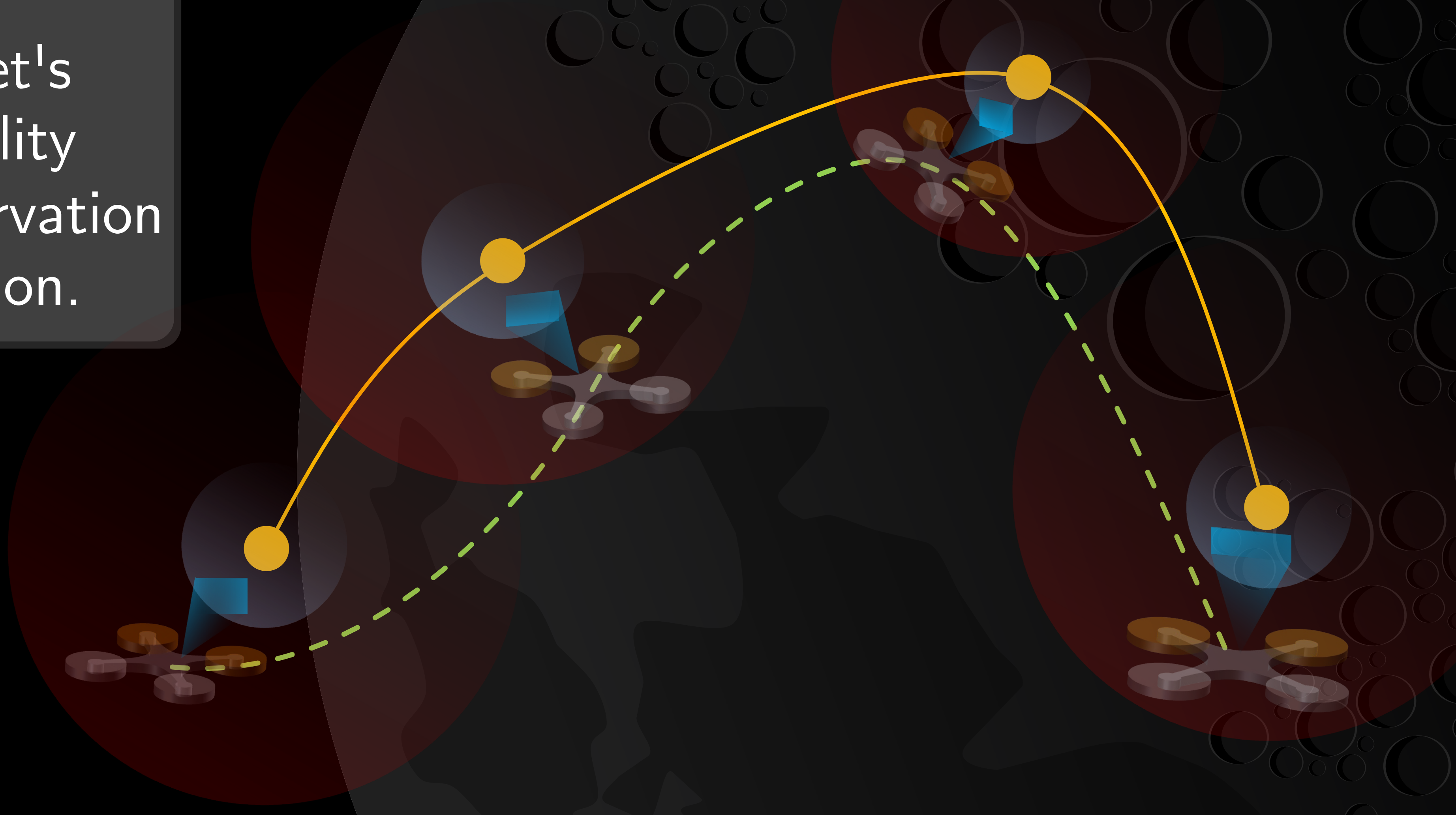
Main Takeaway

Continuous prevents LoS violation better than discrete. However, it sacrifices objective performance

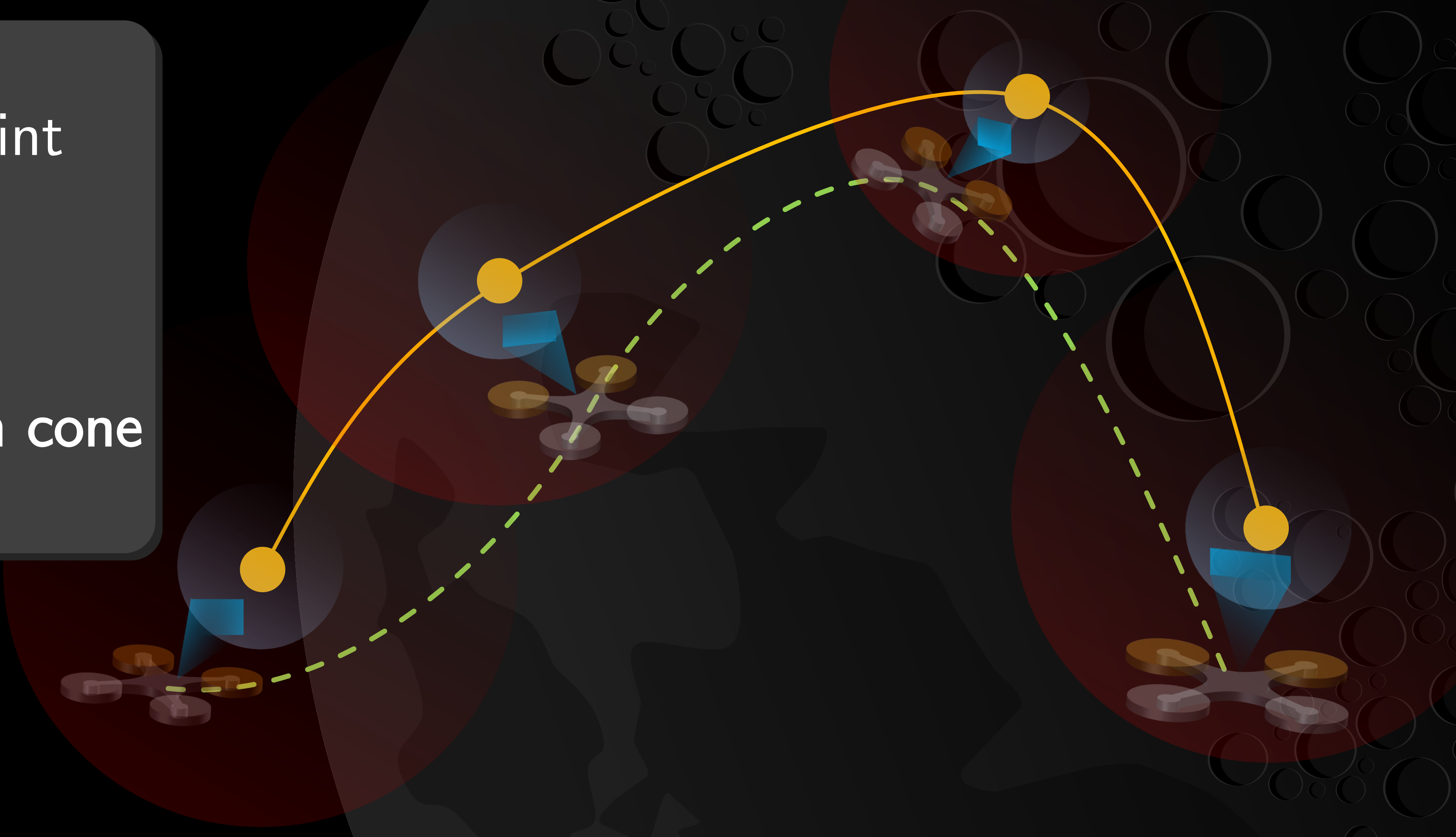
Exploitation: Dynamic Target Tracking

Dynamic target tracking also demonstrates how exploiting a target's kinematic info depends on the visibility model to ensure the persistent observation needed for accurate motion estimation.



Dynamic Target Tracking

Objective: Minimum Fuel
Constraints: Line-of-sight on keypoint
Min-Max range
Boundary constraints
Box constraints
Viewcone: Non-symmetric ∞-Norm cone
Number of Keypoints: 1



Dynamic Target Tracking: Augmented Fuel Dynamics

[Hayner 2025]

We can augment the 6-DoF rigid body dynamics with the 2-Norm of fuel to model the problem in the Mayer form.

Dynamics

Position: $$\dot{r}_{\mathcal{I}_3}(t) = v_{\mathcal{I}_3}(t),$$

Velocity: $$\dot{v}_{\mathcal{I}_3}(t) = \frac{1}{m} C(q_{\mathcal{B}_3 \to \mathcal{I}_3}(t)) T_{\mathcal{B}_3}(t) + g_{\mathcal{I}_3},$$

Attitude: $$\dot{q}_{\mathcal{B}_3 \to \mathcal{I}_3}(t) = \frac{1}{2} \Omega(\omega_{\mathcal{B}_3}(t)) q_{\mathcal{B}_3 \to \mathcal{I}_3}(t),$$

Angular Rate: $$\dot{\omega}_{\mathcal{B}_3}(t) = J_{\mathcal{B}_3}^{-1} \left(M_{\mathcal{B}_3}(t) - [\omega_{\mathcal{B}_3}(t)]_\times J_{\mathcal{B}_3} \omega_{\mathcal{B}_3}(t) \right),$$

Fuel: $$\dot{f}(t) = \left\| \begin{bmatrix} T_{\mathcal{B}_3}(t) & M_{\mathcal{B}_3}(t) \end{bmatrix} \right\|_2$$

Dynamic Target Tracking: Problem Form

$$
\begin{aligned}
\min_{r_{\mathcal{I}_3}, v_{\mathcal{I}_3}, q_{\mathcal{B}_3\to\mathcal{I}_3}, \omega_{\mathcal{B}_3}, f, T_{\mathcal{B}_3}, M_{\mathcal{B}_3}, t_f} \quad & f(t_f) \\
\text{subject to} \quad & r_{\mathcal{I}_3}(0) = r^0, \quad v_{\mathcal{I}_3}(0) = v^0, \\
& q_{\mathcal{B}_3\to\mathcal{I}_3}(0) = q^0, \quad \omega_{\mathcal{B}_3}(0) = \omega^0 \\
& f(0) = 0 \\
& \dot{x} = f_{\text{6DoF}}(t, x(t), u(t)) \\
& \dot{f}(t) = \left\| \begin{bmatrix} T_{\mathcal{B}_3}(t) & M_{\mathcal{B}_3}(t) \end{bmatrix} \right\|_2 \\
& g_{\text{LoS}}(p_{\mathcal{I}_3}(t), r_{\mathcal{I}_3}(t), q_{\mathcal{B}_3\to\mathcal{I}_3}(t), \alpha) \le 0, \\
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& 0 \le f(t) \le f_{\max}, \quad T_{\min} \le T_{\mathcal{B}_3}(t) \le T_{\max}, \\
& M_{\min} \le M_{\mathcal{B}_3}(t) \le M_{\max},
\end{aligned}
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where constraints without explicit time indices hold for all $t \in [0, t_f]$.

[Hayner 2025]

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[Hayner 2025]

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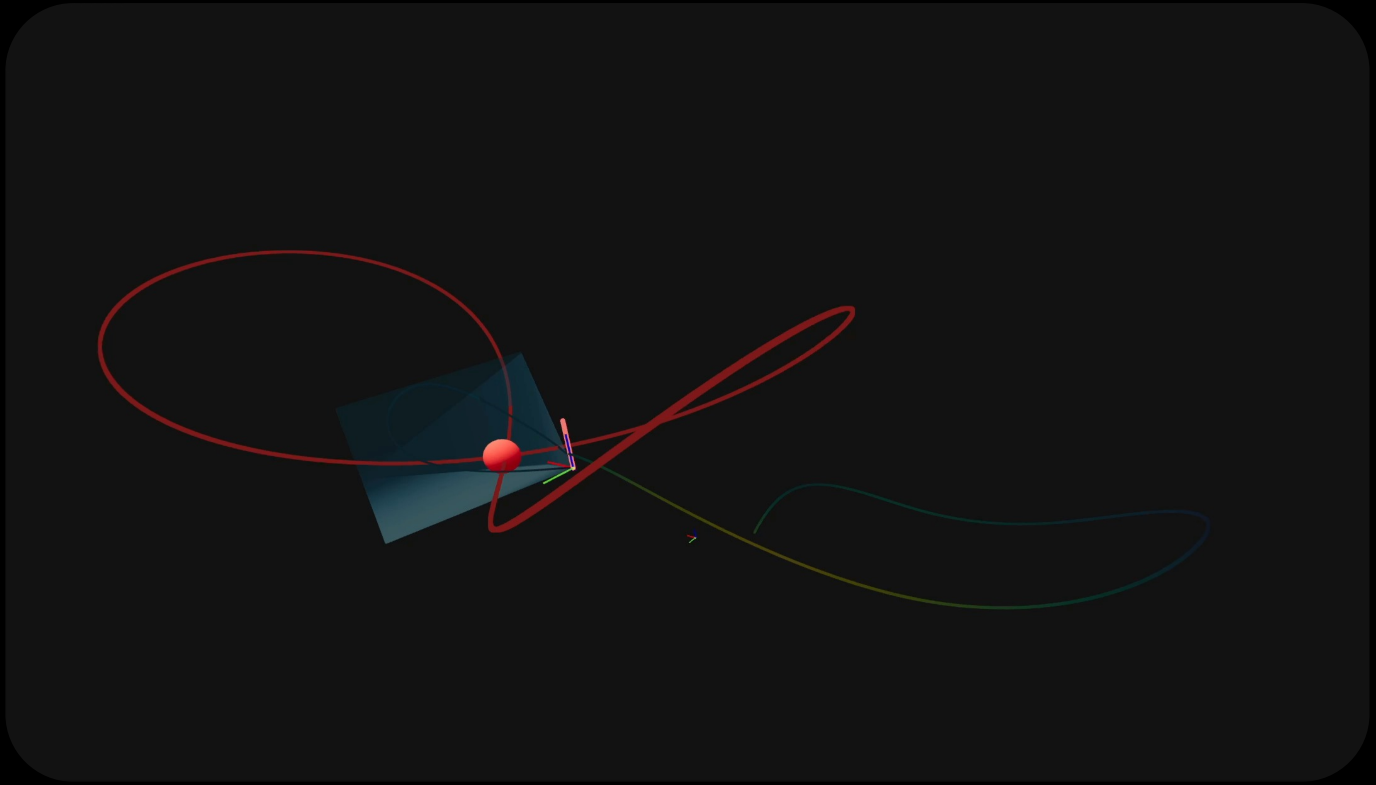

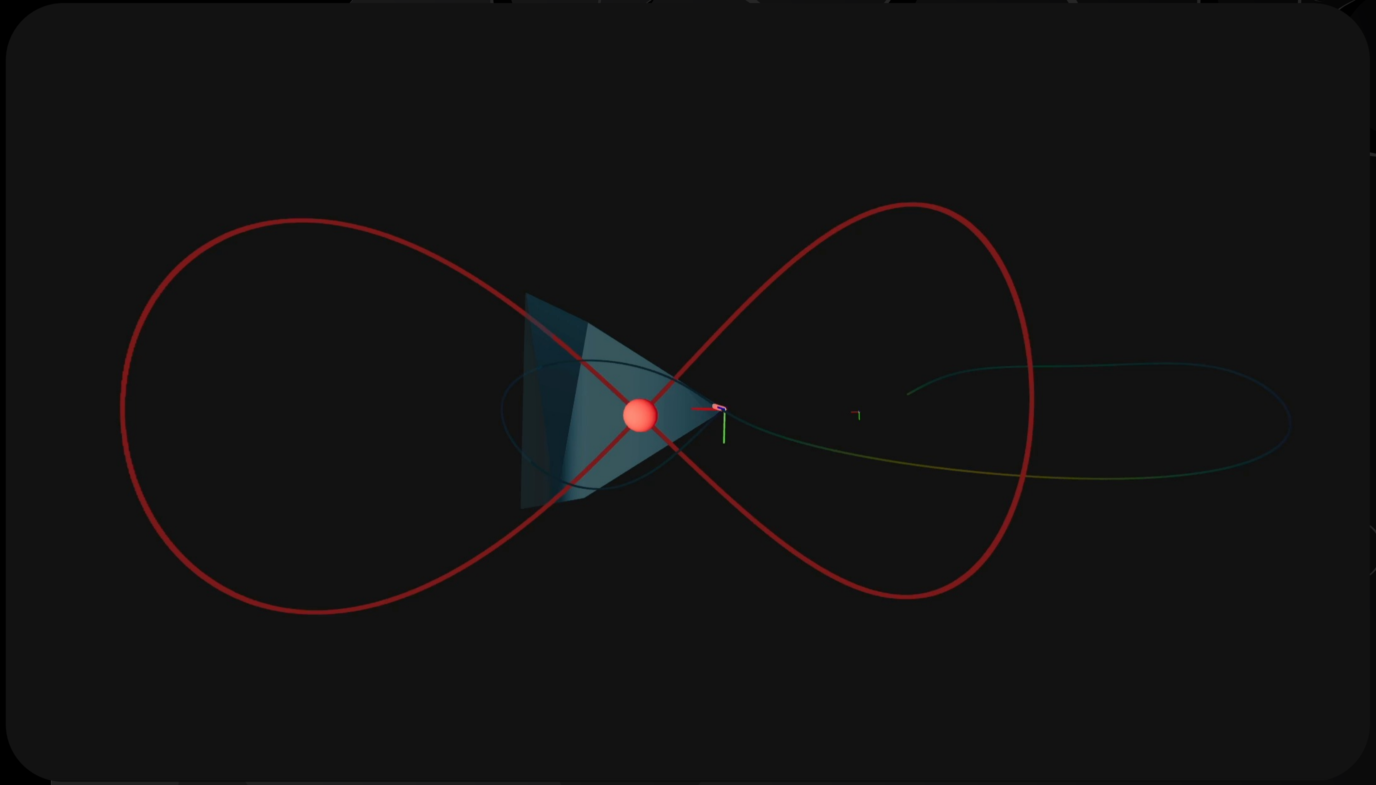

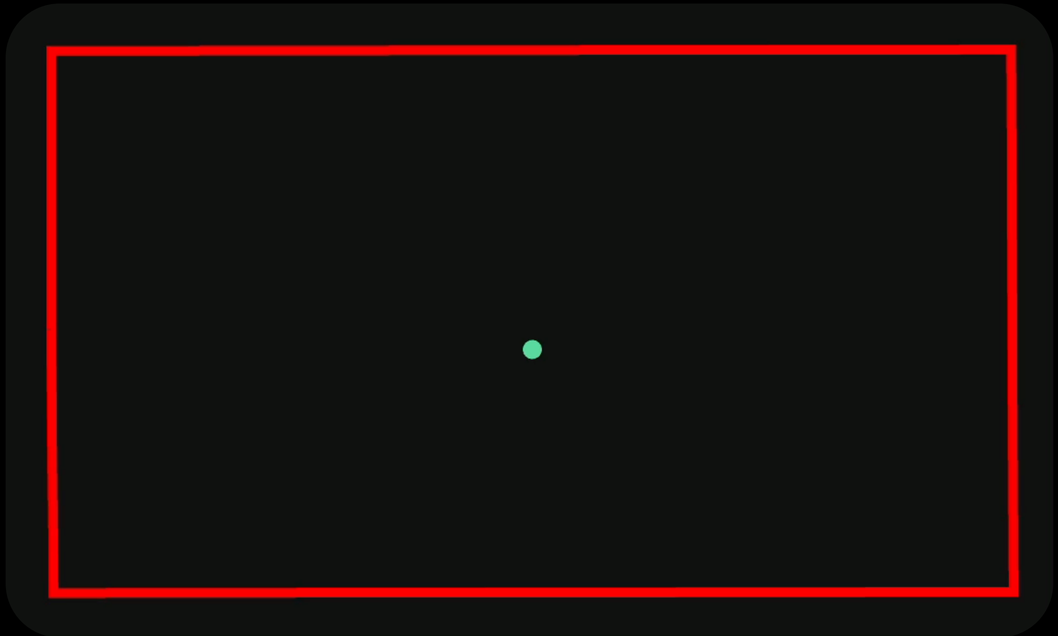

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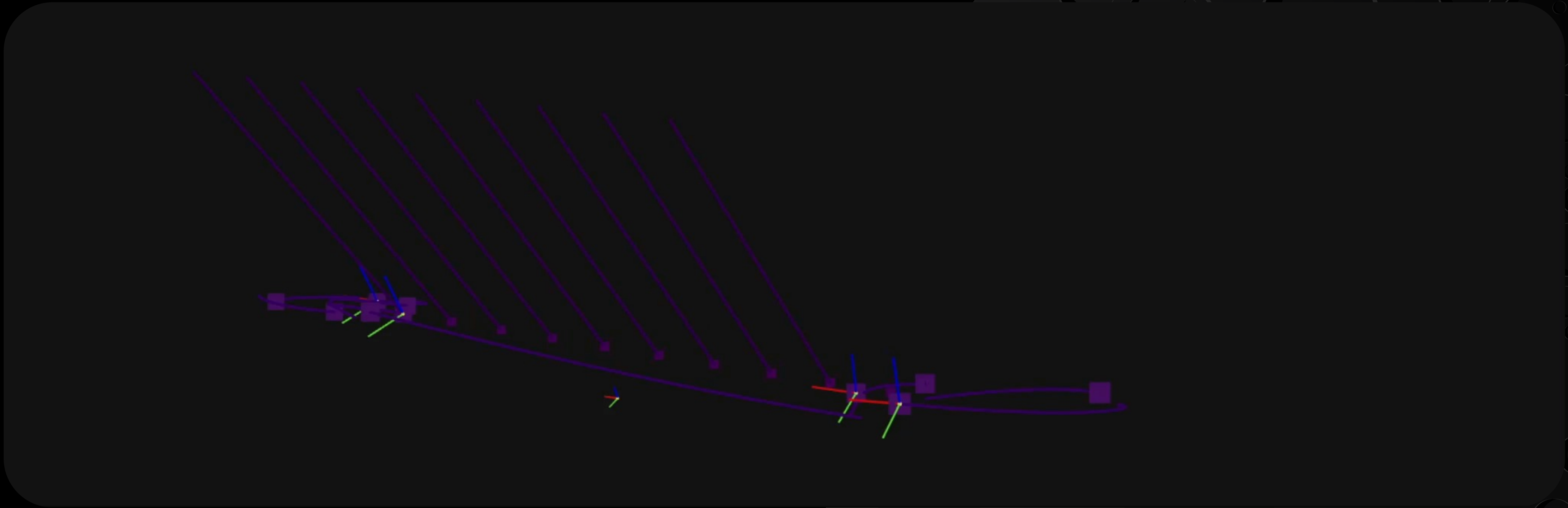
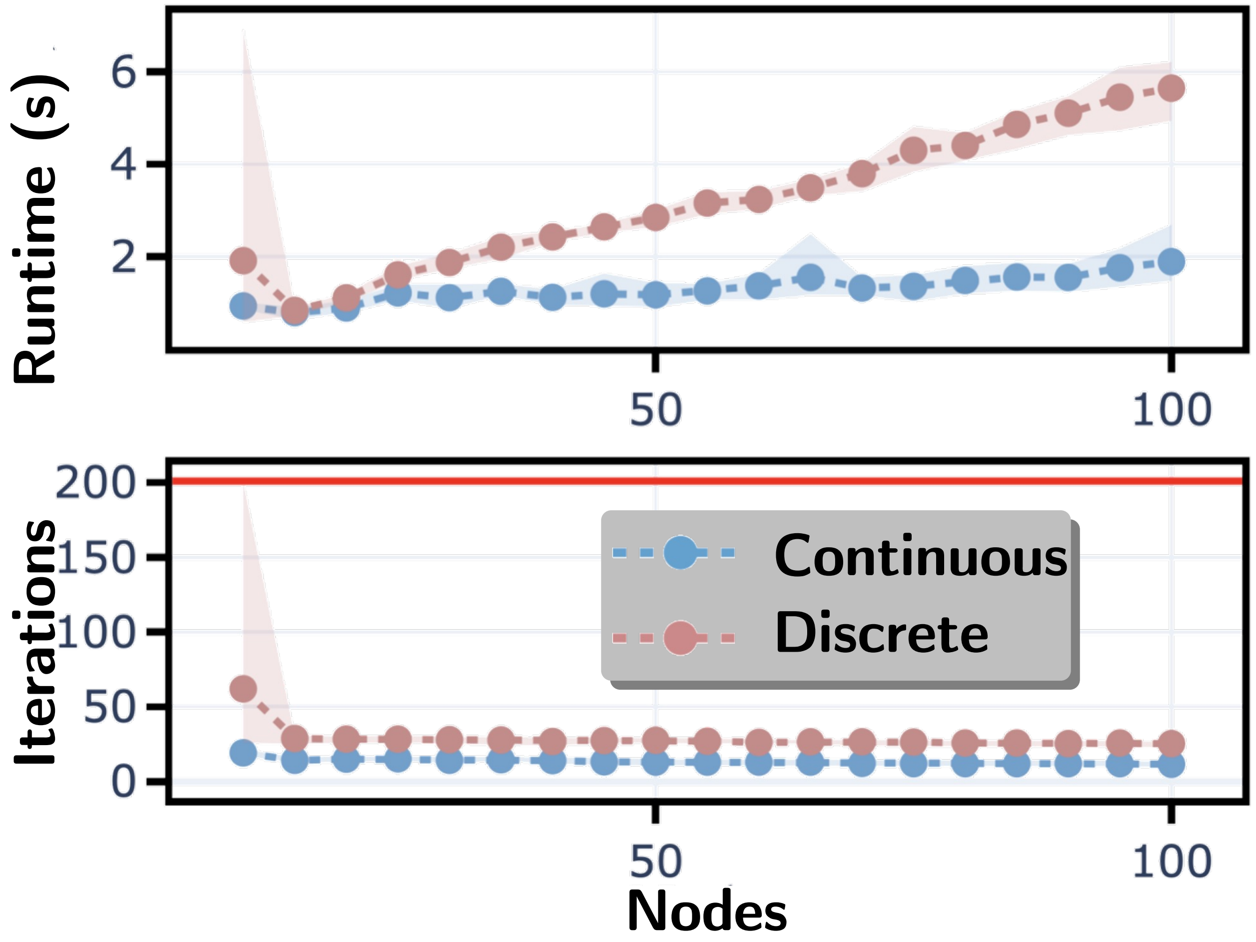


Total Fuel: 364.72
Iterations: 17
Solve Time: 0.316s

[Hayner 2025]

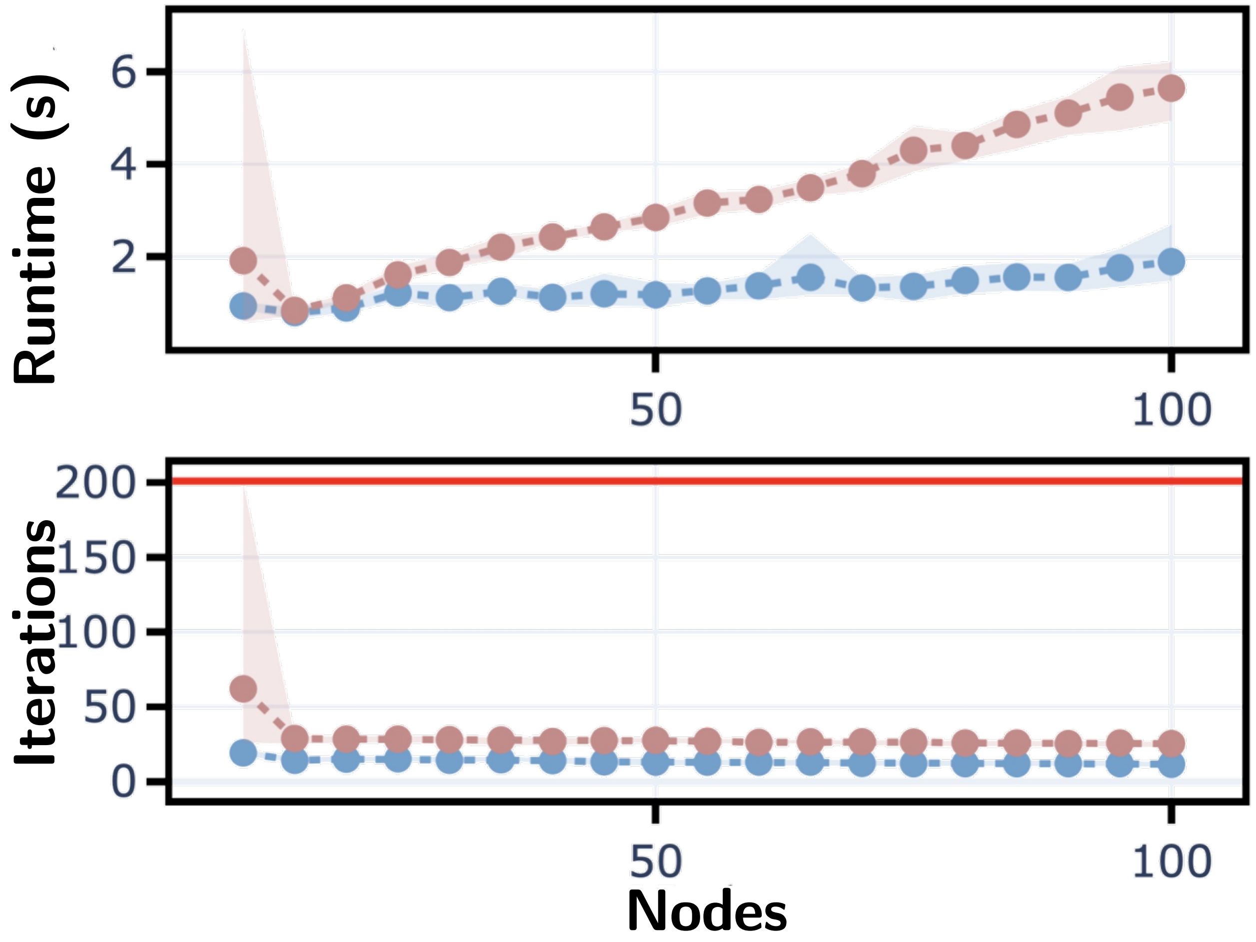
Dynamic Target Tracking: Quantitative Results

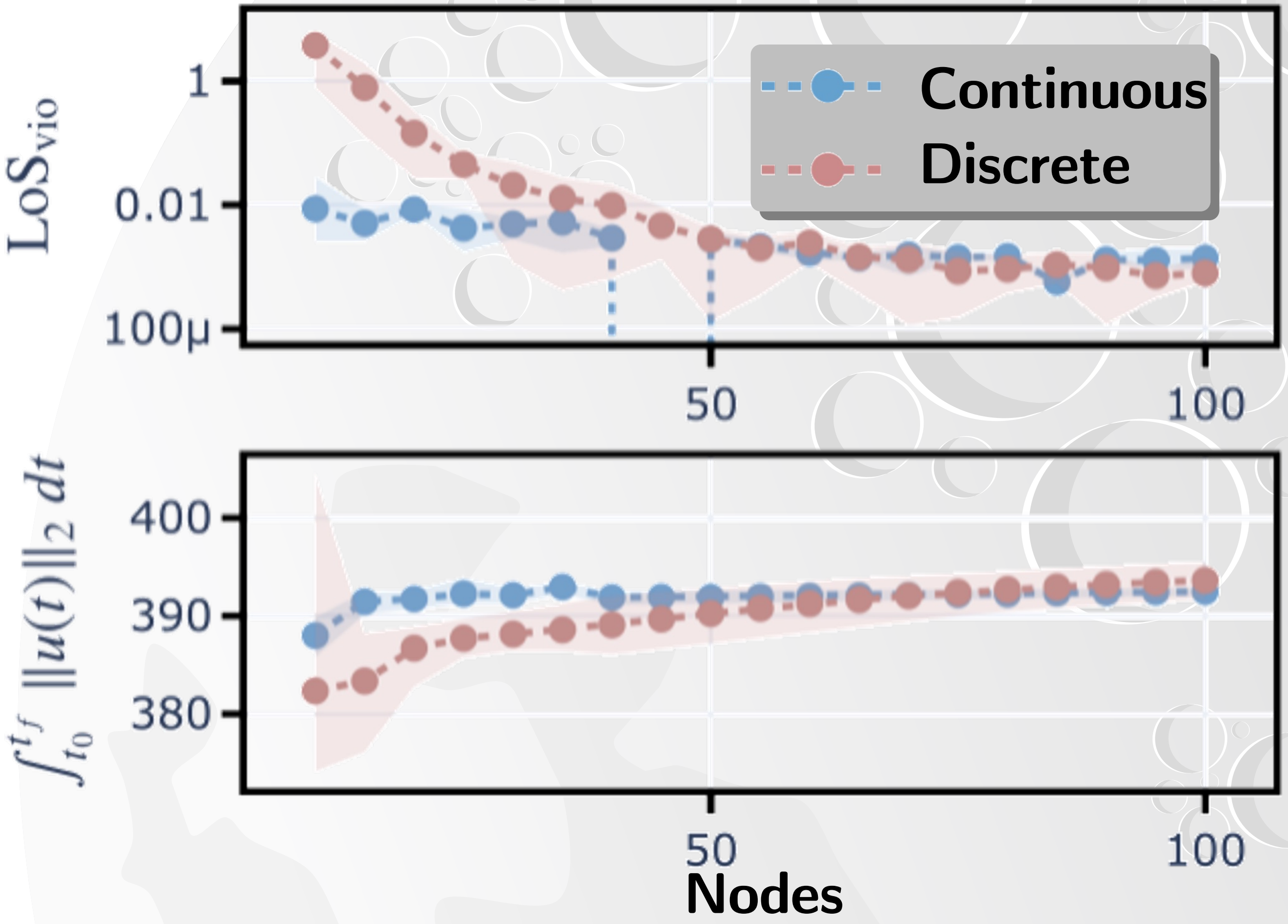


Main Takeaway

Continuous scales better with respect to the discretization grid size.

Dynamic Target Tracking: Quantitative Results





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Exploitation Takeaways

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> ***A2.*** *Discrete better objective performance as it inherently solves a less constrained approximation of the original nonconvex problem*

Q3. How does Continuous scale as the problem size increase?

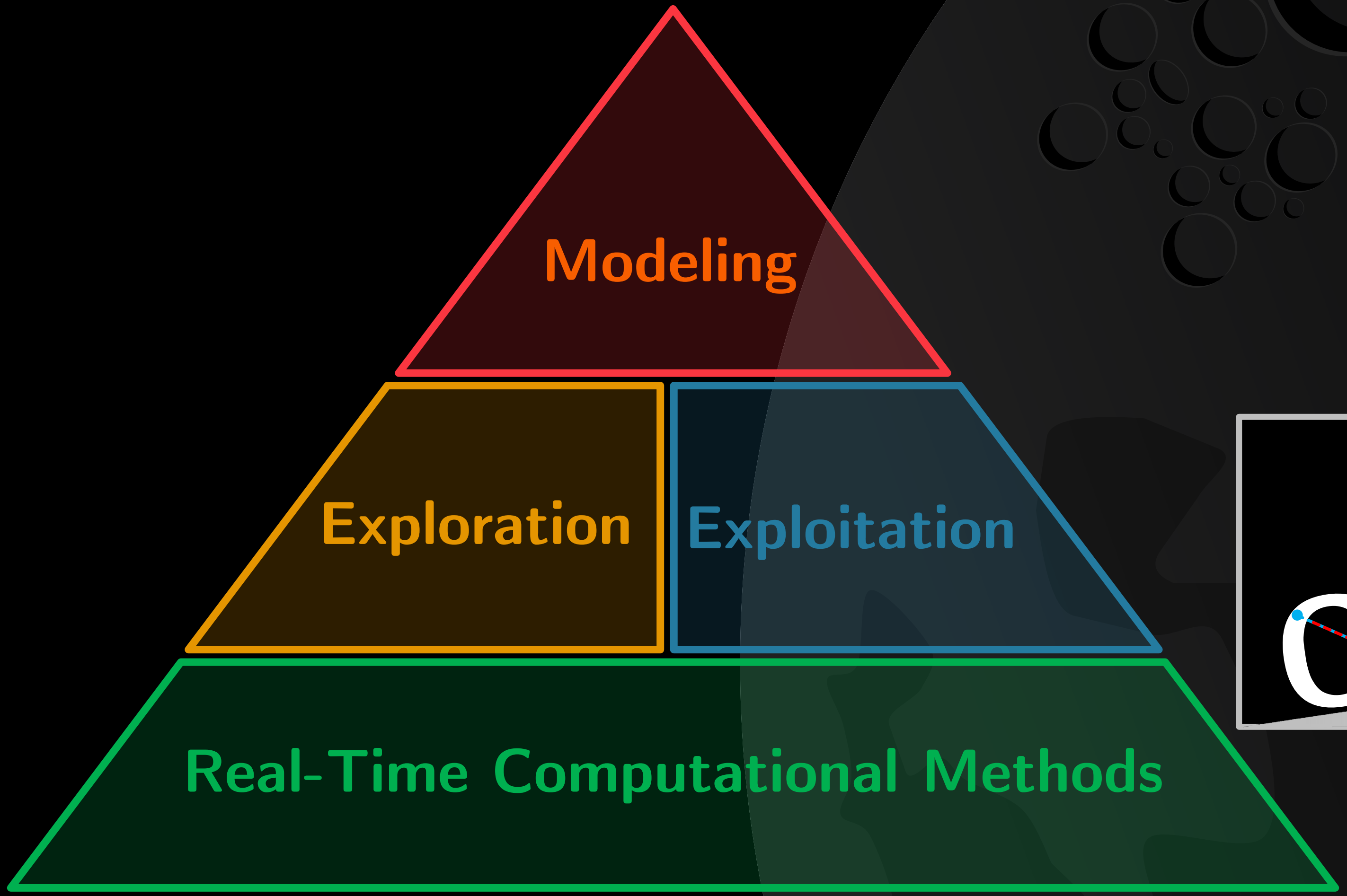
> ***A3.*** *As the discretization grid size increases, Continuous is significantly less affected than Discrete as the convex subproblem has significantly fewer nodal constraints*

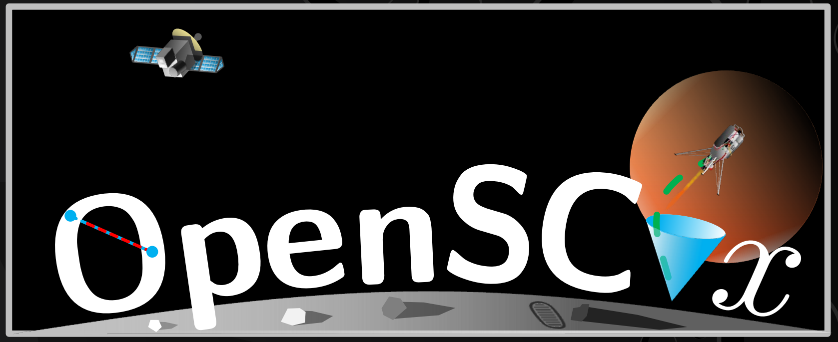
Exploitation

Key Contribution: These formulations jointly optimize the full 6-DoF dynamics under line-of-sight constraints while avoiding differential flatness and decoupled planning limitations.

[Hayner 2025]

Roadmap: Real-Time Computational Methods





OpenSCvx: Requirements

An effective general nonconvex trajectory optimization software should include:

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An effective general nonconvex trajectory optimization software should include:

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R3. Real-time performance,

R4. Agnostic to hardware backend.

OpenSCvx Comparison

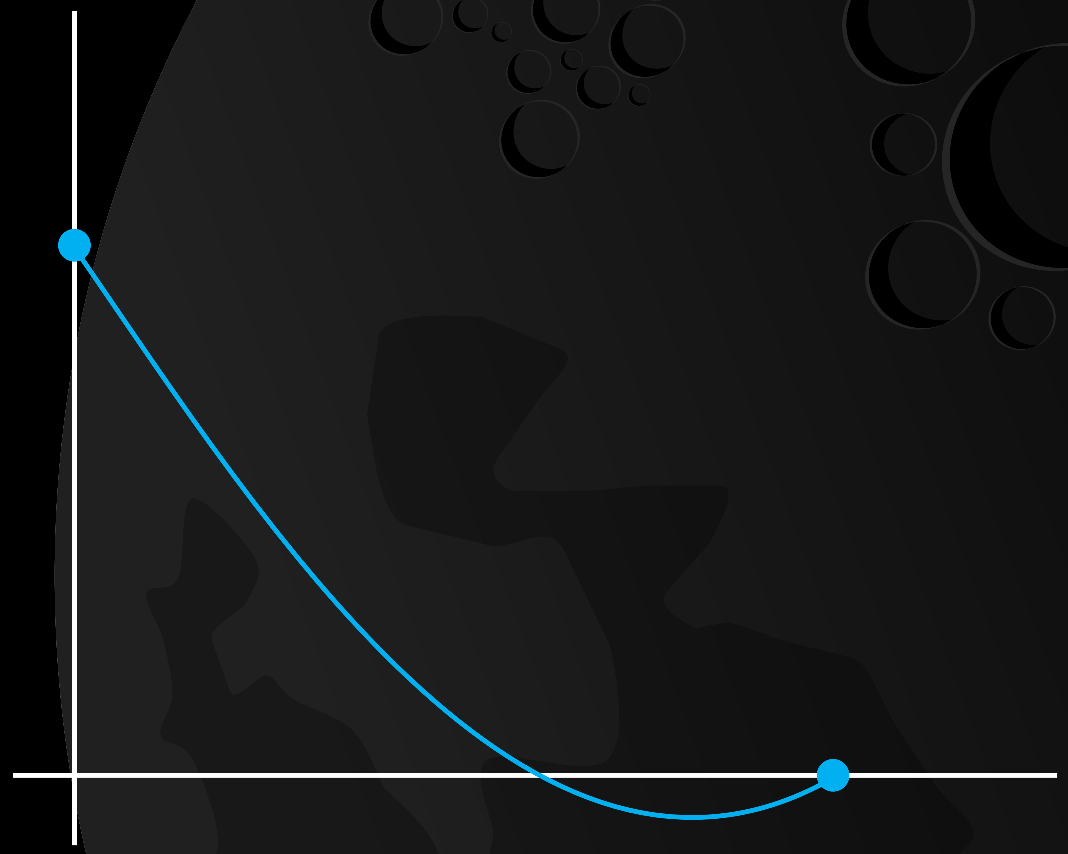
R1. Easy for non-expert users to express problems,
R2. Handle the largest set of problems as possible,
R3. Real-time performance,
R4. Agnostic to hardware backend.

	R1	R2	R3	R4	Availability
OpenSCvx	✓	✓	✓	✓	Open source
SCvxGEN	✓	✓	✓	✗	Unreleased
SCP(t)	✗	✗	✓	✗	Open source
	✗	✗	✓	✓	Open source
GPOPS-ii	✓	✓	✗	✗	Paid

OpenSCvx: Problem Expression

Brachistochrone Problem

$$
\begin{aligned}
\min_{r,v,\theta,t} \quad & t_f \\
\text{s.t.} \quad & r(t_0) = r_0, r(t_f) = r_f \\
& v(t_0) = v_0 \\
& \dot{r}_x = v\sin(\theta) \\
& \dot{r}_y = -v\cos(\theta) \\
& \dot{v} = g\cos(\theta) \\
& r_{\min} \le r \le r_{\max} \\
& v_{\min} \le v \le v_{\max} \\
& \theta_{\min} \le \theta \le \theta_{\max}
\end{aligned}
$$



OpenSCvx: Problem Expression

$$
\begin{aligned}
\min_{r,v,\theta,t} \quad & t_f \\
\text{s.t.} \quad & r(t_0) = r_0, r(t_f) = r_f \\
& v(t_0) = v_0 \\
& \dot{r}_x = v\sin(\theta) \\
& \dot{r}_y = -v\cos(\theta) \\
& \dot{v} = g\cos(\theta) \\
& r_{\min} \le r \le r_{\max} \\
& v_{\min} \le v \le v_{\max} \\
& \theta_{\min} \le \theta \le \theta_{\max}
\end{aligned}
$$

State, Control, and Time Instantiation

```
r = ox.State("position", shape=(2,))
```

```
v = ox.State("velocity", shape=(1,))
```

OpenSCvx: Problem Expression

$$\min_{r,v,\theta,t} \; t_f$$

$$\text{s.t.} \; r(t_0) = r_0, r(t_f) = r_f$$

$$v(t_0) = v_0$$

$$\dot{r}_x = v \sin(\theta)$$

$$\dot{r}_y = -v \cos(\theta)$$

$$\dot{v} = g \cos(\theta)$$

$$r_{\min} \leq r \leq r_{\max}$$

$$v_{\min} \leq v \leq v_{\max}$$

$$\theta_{\min} \leq \theta \leq \theta_{\max}$$

State, Control, and Time Instantiation

```python
r = ox.State("position", shape=(2,))
r.max = [10.0, 10.0]
r.min = [0.0, 0.0]
```

```python
v = ox.State("velocity", shape=(1,))
v.max = [10.0]
v.min = [0.0]
```

OpenSCvx: Problem Expression

$$
\begin{aligned}
\min_{r,v,\theta,t} \quad & t_f \\
\text{s.t.} \quad & r(t_0) = r_0, r(t_f) = r_f \\
& v(t_0) = v_0 \\
& \dot{r}_x = v\sin(\theta) \\
& \dot{r}_y = -v\cos(\theta) \\
& \dot{v} = g\cos(\theta) \\
& r_{\min} \leq r \leq r_{\max} \\
& v_{\min} \leq v \leq v_{\max} \\
& \theta_{\min} \leq \theta \leq \theta_{\max}
\end{aligned}
$$

State, Control, and Time Instantiation

```python
r = ox.State("position", shape=(2,))
r.max = [10.0, 10.0]
r.min = [0.0, 0.0]
r.initial = [0.0, 10.0]
r.final = [10.0, 5.0]


v = ox.State("velocity", shape=(1,))
v.max = [10.0]
v.min = [0.0]
v.initial = [0.0]
v.final = [ox.Free(10.0)]
```

OpenSCvx: Problem Expression

$$
\begin{aligned}
\min_{r,v,\theta,t} \quad & t_f \\
\text{s.t.} \quad & r(t_0) = r_0, r(t_f) = r_f \\
& v(t_0) = v_0 \\
& \dot{r}_x = v\sin(\theta) \\
& \dot{r}_y = -v\cos(\theta) \\
& \dot{v} = g\cos(\theta) \\
& r_{\min} \leq r \leq r_{\max} \\
& v_{\min} \leq v \leq v_{\max} \\
& \theta_{\min} \leq \theta \leq \theta_{\max}
\end{aligned}
$$

State, Control, and Time Instantiation

```
theta = ox.Control("theta", shape=(1,))
```

OpenSCvx: Problem Expression

$$
\begin{aligned}
\min_{r,v,\theta,t} \quad & t_f \\
\text{s.t.} \quad & r(t_0) = r_0, r(t_f) = r_f \\
& v(t_0) = v_0 \\
& \dot{r}_x = v \sin(\theta) \\
& \dot{r}_y = -v \cos(\theta) \\
& \dot{v} = g \cos(\theta) \\
& r_{\min} \leq r \leq r_{\max} \\
& v_{\min} \leq v \leq v_{\max} \\
& \theta_{\min} \leq \theta \leq \theta_{\max}
\end{aligned}
$$

State, Control, and Time Instantiation

```
theta = ox.Control("theta", shape=(1,))
theta.max = [100.5]
theta.min = [0.0]
```

OpenSCvx: Problem Expression

$$
\begin{aligned}
\min_{r,v,\theta,t} \quad & t_f \\
\text{s.t.} \quad & r(t_0) = r_0, r(t_f) = r_f \\
& v(t_0) = v_0 \\
& \dot{r}_x = v \sin(\theta) \\
& \dot{r}_y = -v \cos(\theta) \\
& \dot{v} = g \cos(\theta) \\
& r_{\min} \leq r \leq r_{\max} \\
& v_{\min} \leq v \leq v_{\max} \\
& \theta_{\min} \leq \theta \leq \theta_{\max}
\end{aligned}
$$

State, Control, and Time Instantiation

```
theta = ox.Control("theta", shape=(1,))
theta.max = [100.5]
theta.min = [0.0]
theta.guess = linspace(5, 100.5, n)
```

OpenSCvx: Problem Expression

$$
\begin{aligned}
\min_{r,v,\theta,t} \quad & t_f \\
\text{s.t.} \quad & r(t_0) = r_0, r(t_f) = r_f \\
& v(t_0) = v_0 \\
& \dot{r}_x = v \sin(\theta) \\
& \dot{r}_y = -v \cos(\theta) \\
& \dot{v} = g \cos(\theta) \\
& r_{\min} \leq r \leq r_{\max} \\
& v_{\min} \leq v \leq v_{\max} \\
& \theta_{\min} \leq \theta \leq \theta_{\max}
\end{aligned}
$$

State, Control, and Time Instantiation

```
theta = ox.Control("theta", shape=(1,))
theta.max = [100.5]
theta.min = [0.0]
theta.guess = linspace(5, 100.5, n)


time = ox.Time()
```

OpenSCvx: Problem Expression

$$
\begin{aligned}
\min_{r,v,\theta,t} \quad & t_f \\
\text{s.t.} \quad & r(t_0) = r_0, r(t_f) = r_f \\
& v(t_0) = v_0 \\
& \dot{r}_x = v\sin(\theta) \\
& \dot{r}_y = -v\cos(\theta) \\
& \dot{v} = g\cos(\theta) \\
& r_{\min} \le r \le r_{\max} \\
& v_{\min} \le v \le v_{\max} \\
& \theta_{\min} \le \theta \le \theta_{\max}
\end{aligned}
$$

State, Control, and Time Instantiation

```python
theta = ox.Control("theta", shape=(1,))
theta.max = [100.5]
theta.min = [0.0]
theta.guess = linspace(5, 100.5, n)


time = ox.Time()
time.initial = 0.0
time.final = ox.Minimize(total_time)
```

OpenSCvx: Problem Expression

$$
\begin{aligned}
\min_{r,v,\theta,t} \quad & t_f \\
\text{s.t.} \quad & r(t_0) = r_0, r(t_f) = r_f \\
& v(t_0) = v_0 \\
& \dot{r}_x = v \sin(\theta) \\
& \dot{r}_y = -v \cos(\theta) \\
& \dot{v} = g \cos(\theta) \\
& r_{\min} \leq r \leq r_{\max} \\
& v_{\min} \leq v \leq v_{\max} \\
& \theta_{\min} \leq \theta \leq \theta_{\max}
\end{aligned}
$$

State, Control, and Time Instantiation

```
theta = ox.Control("theta", shape=(1,))
theta.max = [100.5]
theta.min = [0.0]
theta.guess = linspace(5, 100.5, n)


time = ox.Time()
time.initial = 0.0
time.final = ox.Minimize(total_time)
time.min = 0.0
time.max = total_time
```

OpenSCvx: Problem Expression

$$
\begin{aligned}
\min_{r,v,\theta,t} \quad & t_f \\
\text{s.t.} \quad & r(t_0) = r_0, r(t_f) = r_f \\
& v(t_0) = v_0 \\
& \dot{r}_x = v \sin(\theta) \\
& \dot{r}_y = -v \cos(\theta) \\
& \dot{v} = g \cos(\theta) \\
& r_{\min} \leq r \leq r_{\max} \\
& v_{\min} \leq v \leq v_{\max} \\
& \theta_{\min} \leq \theta \leq \theta_{\max}
\end{aligned}
$$

Dynamics

```
dynamics = {
        "position": ox.Concat(
                velocity[0] * ox.Sin(theta[0]),
                -velocity[0] * ox.Cos(theta[0]),
                )
}
```

OpenSCvx: Problem Expression

$$
\begin{aligned}
\min_{r,v,\theta,t} \quad & t_f \\
\text{s.t.} \quad & r(t_0) = r_0, r(t_f) = r_f \\
& v(t_0) = v_0 \\
& \dot{r}_x = v\sin(\theta) \\
& \dot{r}_y = -v\cos(\theta) \\
& \dot{v} = g\cos(\theta) \\
& r_{\min} \le r \le r_{\max} \\
& v_{\min} \le v \le v_{\max} \\
& \theta_{\min} \le \theta \le \theta_{\max}
\end{aligned}
$$

Dynamics

```
dynamics = {
        "position": ox.Concat(
                velocity[0] * ox.Sin(theta[0]),
                -velocity[0] * ox.Cos(theta[0]),
                ),
        "velocity": g * ox.Cos(theta[0]),
}
```

OpenSCvx: Problem Expression

$$\min_{r,v,\theta,t} \; t_f$$

$$\text{s.t.} \; r(t_0) = r_0, r(t_f) = r_f$$

$$v(t_0) = v_0$$

$$\dot{r}_x = v\sin(\theta)$$

$$\dot{r}_y = -v\cos(\theta)$$

$$\dot{v} = g\cos(\theta)$$

$$r_{\min} \le r \le r_{\max}$$

$$v_{\min} \le v \le v_{\max}$$

$$\theta_{\min} \le \theta \le \theta_{\max}$$

Constraints

```
constraint_exprs = []
for state in states:
    constraint_exprs.extend([ox.ctcs(state <= state.max),
                             ox.ctcs(state.min <= state)])
```

OpenSCvx: Problem Expression

$$
\begin{aligned}
\min_{r,v,\theta,t} \quad & t_f \\
\text{s.t.} \quad & r(t_0) = r_0, r(t_f) = r_f \\
& v(t_0) = v_0 \\
& \dot{r}_x = v \sin(\theta) \\
& \dot{r}_y = -v \cos(\theta) \\
& \dot{v} = g \cos(\theta) \\
& r_{\min} \leq r \leq r_{\max} \\
& v_{\min} \leq v \leq v_{\max} \\
& \theta_{\min} \leq \theta \leq \theta_{\max}
\end{aligned}
$$

Constraints

```
constraint_exprs = []
for state in states:
    constraint_exprs.extend([ox.ctcs(state <= state.max),
                             ox.ctcs(state.min <= state)])
```

By default, the package will apply the box constraint to controls.

OpenSCvx: Problem Expression

While this package focuses on minimizing the number of required inputs, an expert user can selectively override the symbolic expression interface and replace any algorithmic component (e.g. discretization, integration, SCvx) to fit their specific needs.

OpenSCvx: Broad Problem Class Applicability

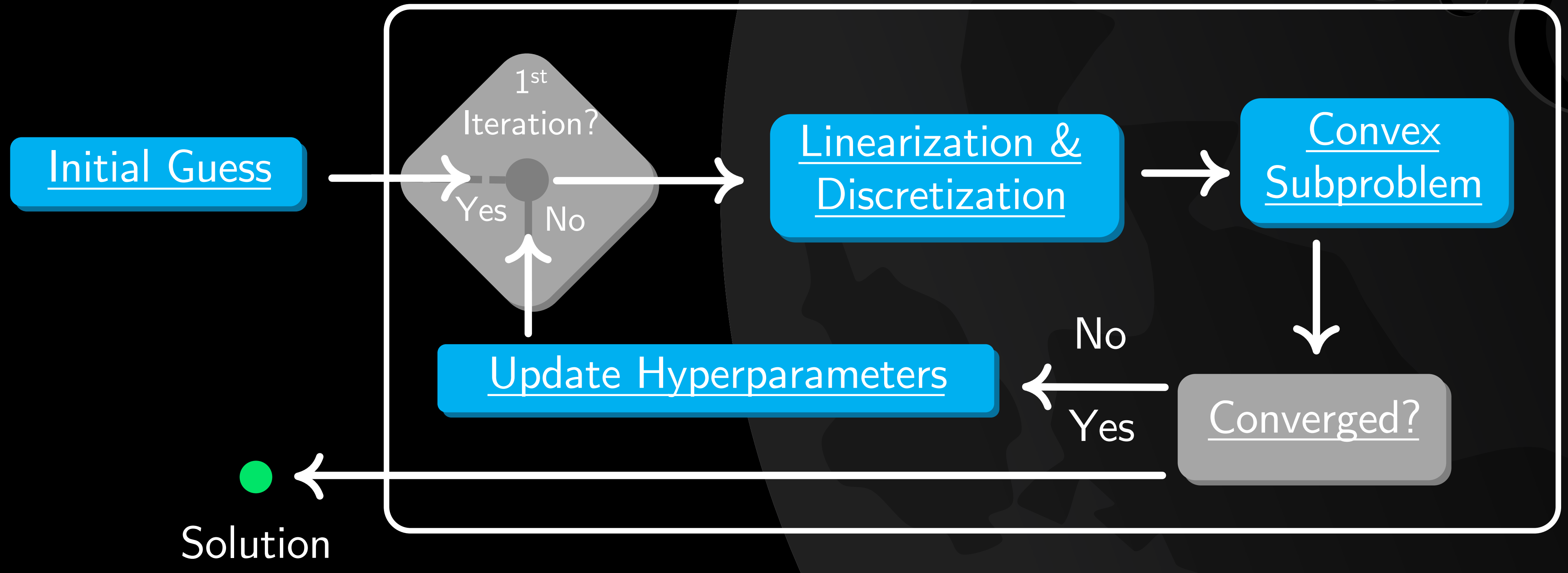
To ensure broad applicability across problem classes, this package utilizes the CT-SCvx algorithm, which solves problems posed in the Mayer Form.

Mayer Form

$$
\begin{aligned}
\min_{x,u,t_f} \quad & y(t_f) + L(t_f, x(t_f)) \\
\text{subject to} \quad & \dot{x}(t) = f(t, x(t), u(t)), \quad t \in [t_0, t_f] \\
& g(t, x(t), u(t)) \leq 0_{n_g}, \quad t \in [t_0, t_f] \\
& h(t, x(t), u(t)) = 0_{n_h}, \quad t \in [t_0, t_f] \\
& P(t_0, x(t_0), t_f, x(t_f)) \leq 0_{n_P} \\
& Q(t_0, x(t_0), t_f, x(t_f)) = 0_{n_Q}
\end{aligned}
$$

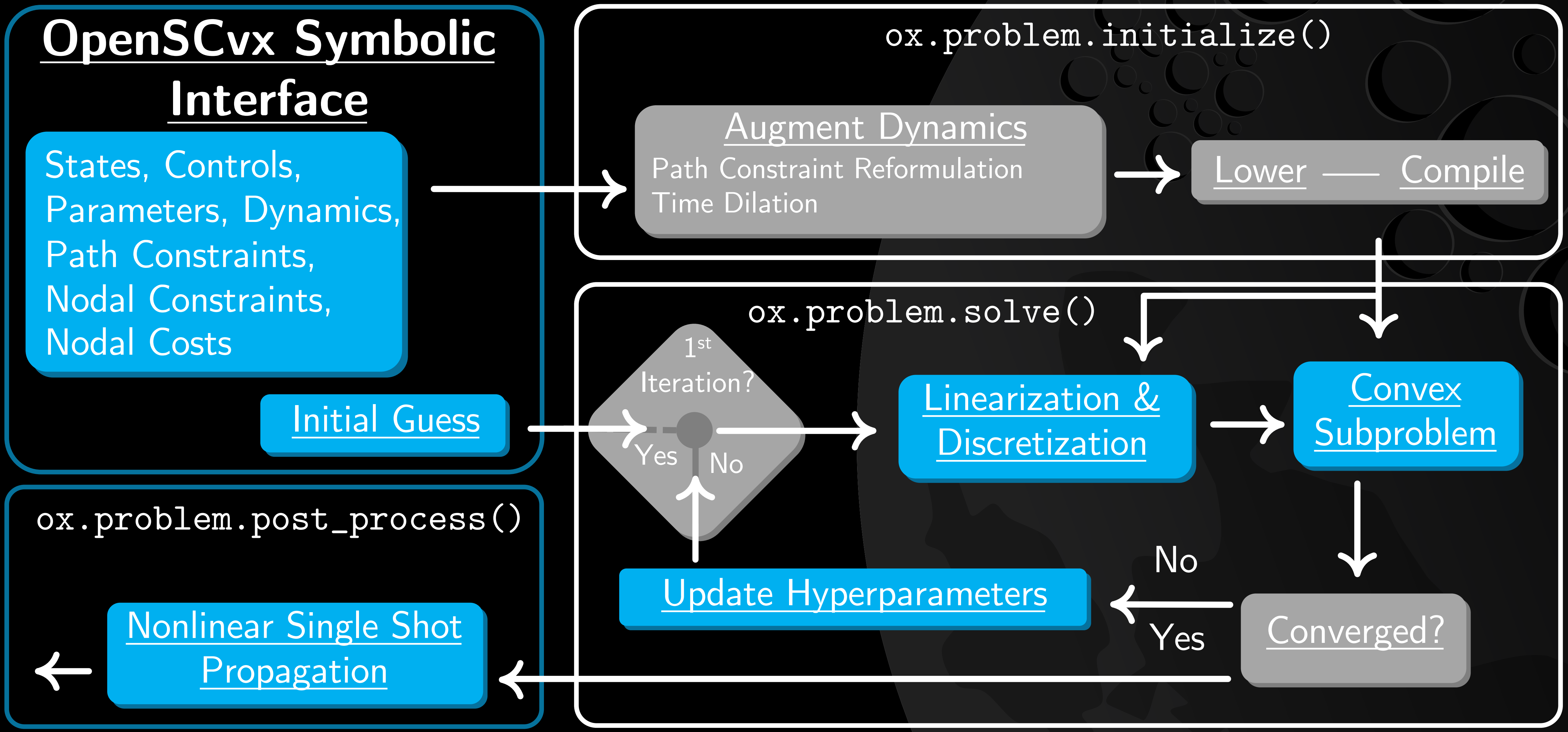
[Liberzon 2011]

OpenSCvx: CT-SCvx



[Elango 2025]

OpenSCvx: Overview



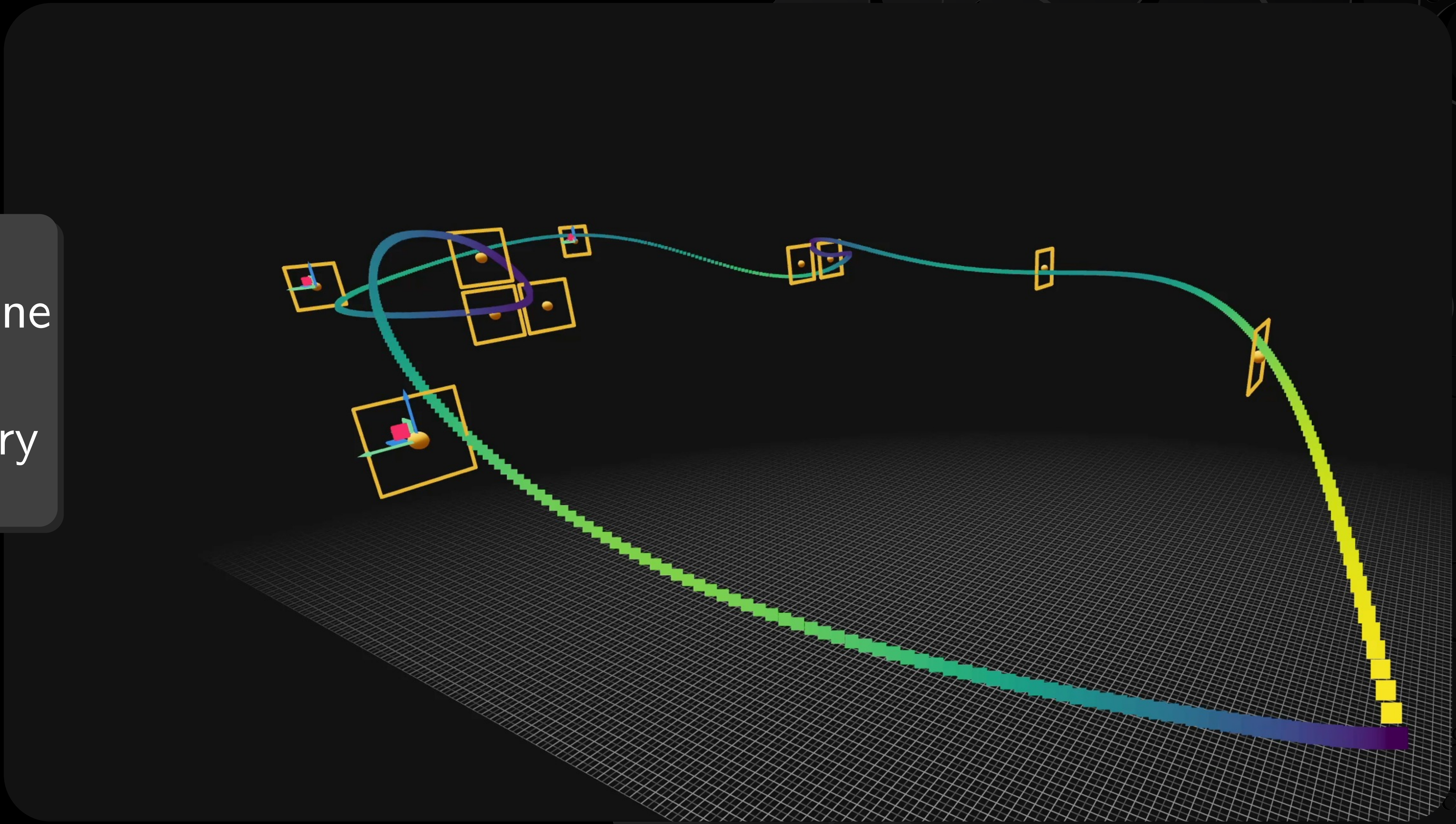
[Hayner 2026]

OpenSCvx: Real-time Performance

Uses Python with a CVXPY and JAX backend to provide fast convex solvers, efficient automated vectorization, and a differentiable linear algebra library.

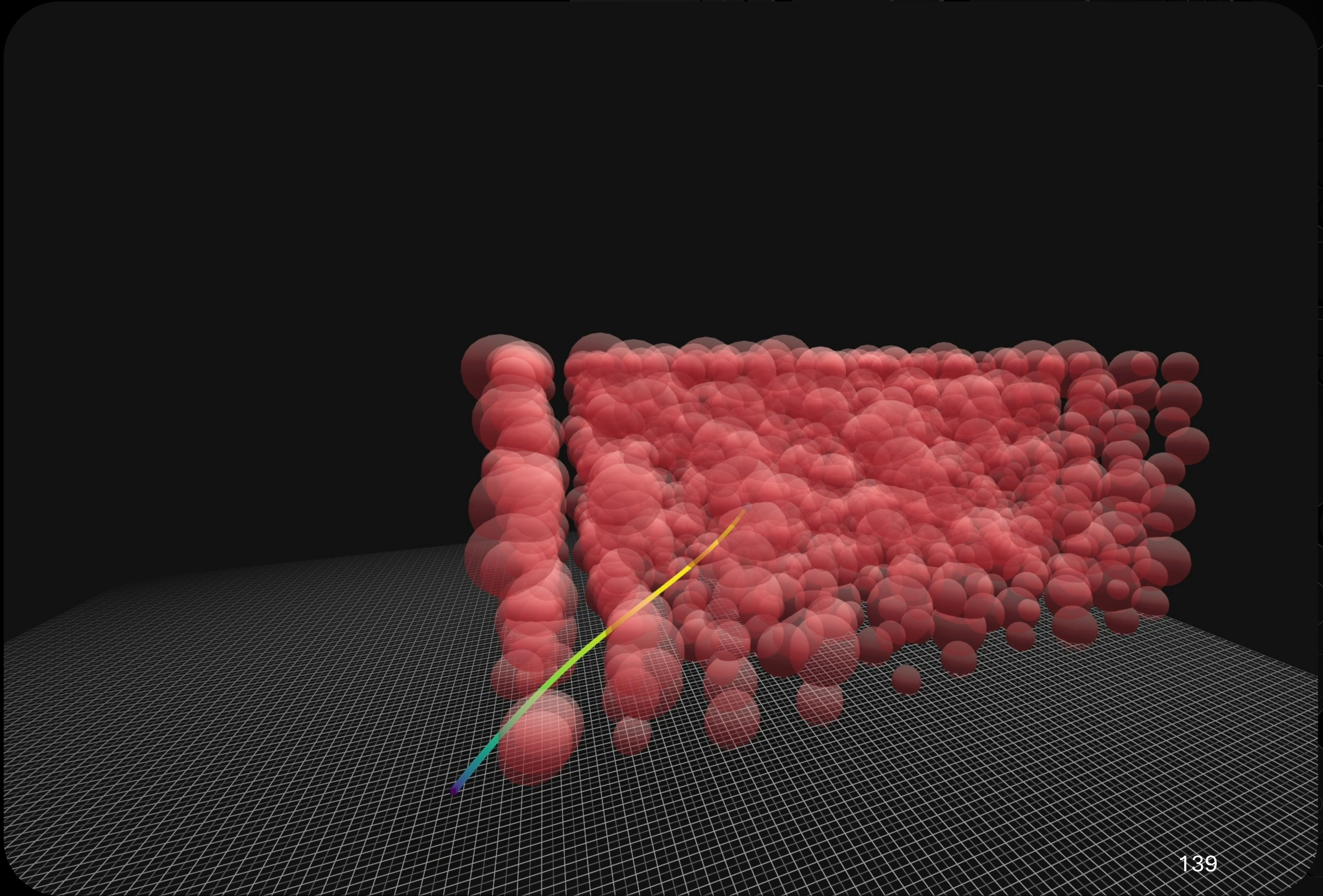
OpenSCvx: Live Real-time Drone Racing

Objective: Min Time
Dynamics: 6DoF Drone
Constraints: Gates
Boundary
Box



OpenSCvx: Vectorized Real-Time Obstacle Avoidance

Time of Flight: 8.26
Iterations: 8
Solve Time: 0.064s
Obstacles: 1000



OpenSCvx: Hardware/Software Agnostic

Since OpenSCvx heavily leverages JAX, we inherit lots of the benefits of being agnostic to the hardware backend, meaning we can seamlessly leverage GPU's, allowing for problems to scale very large while remaining tractable.

Furthermore, JAX is an extremely widely adopted numerical computing engine, backed by Google, with over 800 unique contributors and is a dependency in over 45k repositories on GitHub.

OpenSCvx: Hardware/Software Agnostic

OpenSCvx has been used on:

OpenSCvx

Key Contributions: This package provides a high-performance, scalable, non-convex trajectory optimization solver underneath an intuitive symbolic user interface.

[Hayner 2026]

My Publications

C. R. Hayner, J. M. Carson III, B. Açıkmeşe and K. Leung, "Continuous-Time Line-of-Sight Constrained Trajectory Planning for 6-Degree of Freedom Systems," in *IEEE Robotics and Automation Letters*, vol. 10, no. 5, pp. 4332-4339, May 2025

C. R. Hayner, S. C. Buckner, D. Broyles, E. Madewell, K. Leung and B. Açikmeşe, "HALO: Hazard-Aware Landing Optimization for Autonomous Systems," *2023 IEEE International Conference on Robotics and Automation (ICRA),* London, United Kingdom, 2023, pp. 3261-3267

C. R. Hayner, N. Pavlasek, K. Leung, B. Acikmese and J. M. Carson III. "Information-Aware Powered Descent Guidance for Entry, Descent and Landing" AIAA 2025-1896. *AIAA SCITECH 2025 Forum*. January 2025.

K. Echigo, **C. R. Hayner**, A. Mittal, S. B. Sarsılmaz, M. W. Harris and B. Açıkmeşe, "Linear Programming Approach to Relative-Orbit Control With Element-Wise Quantized Control," in IEEE Control Systems Letters, vol. 7, pp. 3042-3047, 2023

K. Echigo, **C. R. Hayner**, A. Mittal, S. B. Sarsilmaz, M. Harris and B. Acikmese."Convex Trajectory Planning for Proximity Operations using Electric Propulsion with Quantized Thrust," AIAA 2023-0493. *AIAA SCITECH 2023 Forum*. January 2023.

D. Broyles, **C. R. Hayner** and K. Leung, "WiSARD: A Labeled Visual and Thermal Image Dataset for Wilderness Search and Rescue," 2022 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Kyoto, Japan, 2022, pp. 9467-9474

Acknowledgments

I would like to acknowledge my co-author and partner-in-crime for all things OpenSCvx, Griffin Norris as well as all the coauthors and colleagues I have had privilege to work with over the past 4 and a half years in my Ph.D. journey.

Any Questions?

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